

Laboratory work 1

Calculation of Matrix Determinant.

References.

Nonsingular matrix. Gauss method. Diagonal form of matrix. Definition of matrix determinant. Definition of minor, algebraic matrix complement.

Brief theoretical information.

Definition. Square n matrix is nonsingular if its rank equals n .

Definition. Square m matrix $D=(d_{ij})$ is diagonal, if $d_{ij}=0$, $i=1,2,\dots,m$; $j=1,2,\dots,m$, $i \neq j$.

Definition. Any diagonal matrix D , obtained from matrix A by multiplication on elementary matrixes is called Diagonal form of matrix A .

Definition. Product of diagonal elements in any diagonal form of square matrix A is called matrix A determinant.

Definition. Minor k matrix A is a determinant of matrix B order $k \times k$, which consists of selected k rows and columns of matrix A , which are placed in the same degree as in matrix A .

Definition. Algebraic complement of element a_{rs} in square matrix A is a scalar $(-1)^{r+s}b$, where b is minor of $(n-1)$ matrix A , which is obtained by extraction of row $N^{\circ} r$ column $N^{\circ} s$ from it.

Theorem. Matrix determinant equals sum of products of elements of any row (column) of this matrix and their algebraic complements.

 Text book «Linear Algebra» O.V.Spivakovsky, V.A.Kreknin

Algorithm of solution.

Using Gauss method (method of elementary transformations) matrix must be reduced to diagonal (or triangle) view, remembering that permutation of rows or columns changes the sign of determinant. Determinant is obtained as product of diagonal elements and matrix multiplier.

Task. Find matrix determinant

$$(N^{\circ}22) \begin{bmatrix} 4 & 2 & 1 \\ 7 & 8 & 10 \\ 3 & 2 & 8 \end{bmatrix}$$

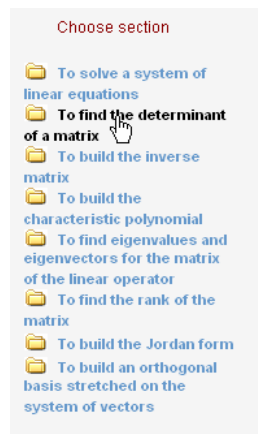
Algorithm of problem loading

1. Execute the following actions to load task in "Task Book":

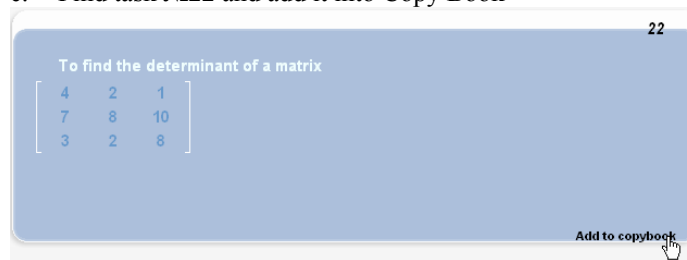
- a. In main menu choose "Task Book":

Main page	Testing	Taskbook	Copybook	Solution environment	Book	Laboratory	Tasks	Report
Discussions	About project	LogOn						

- b. In «Task Book» choose section «Find matrix rank»:



c. Find task №22 and add it into Copy Book

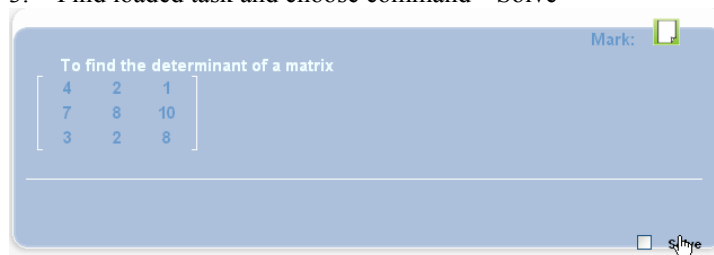


2. Go to Copy Book:

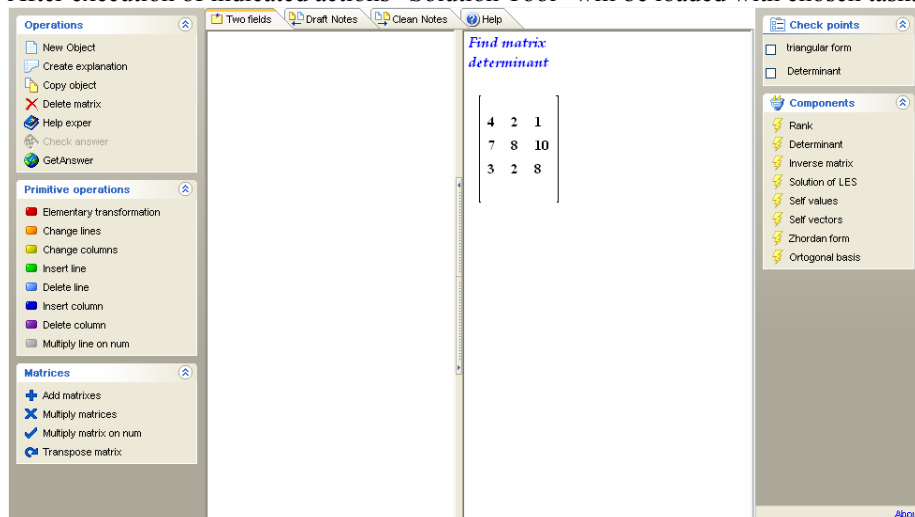
a. In main menu choose command “Copy Book”:



3. Find loaded task and choose command “Solve”



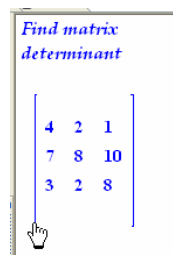
After execution of indicated actions “Solution Tool” will be loaded with chosen task.



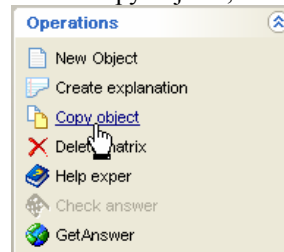
Algorithm of problem solution.

1. Copy matrix to Rough Note-Book:

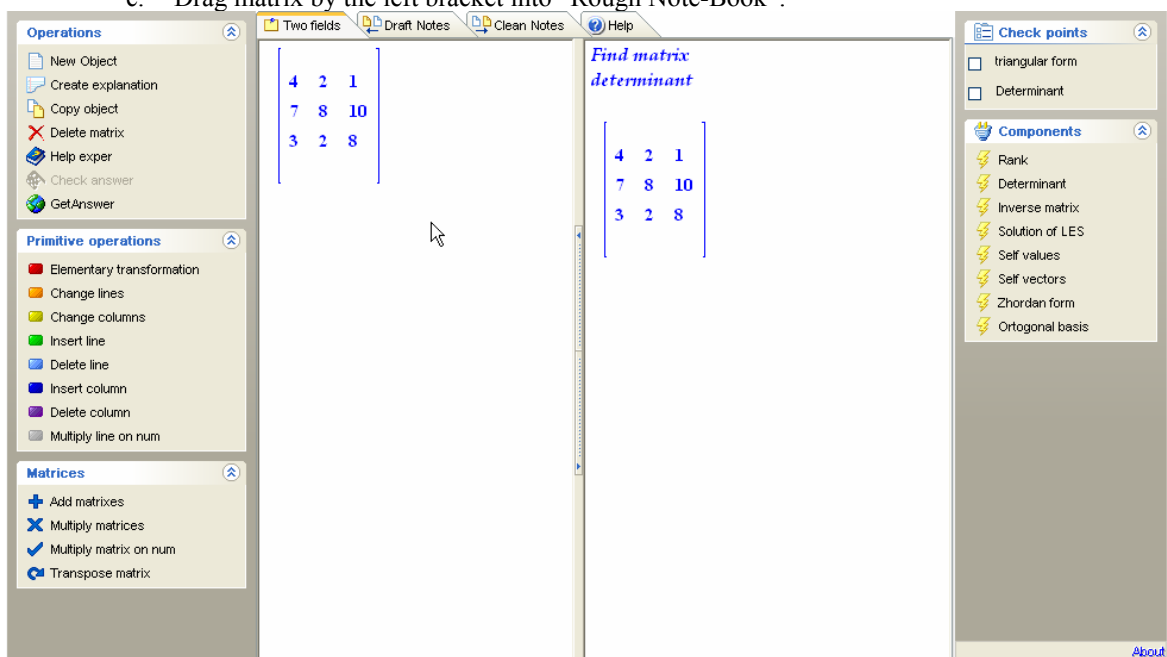
a. Select matrix by clicking on the left bracket;



b. In menu “Operations” choose command “Copy object”;

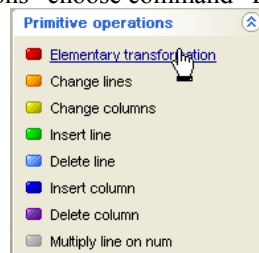


c. Drag matrix by the left bracket into “Rough Note-Book”:



2. Reduce matrix to triangle view:

a. In menu “Elementary transformations” choose command “Elementary transformations”:

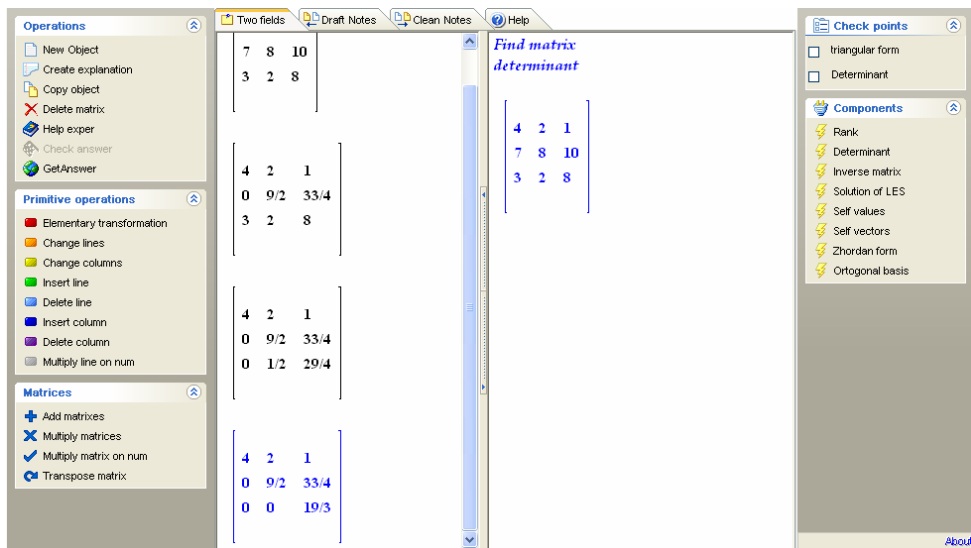


b. Drag the first element of the first matrix line to the first element of the second line of this matrix:



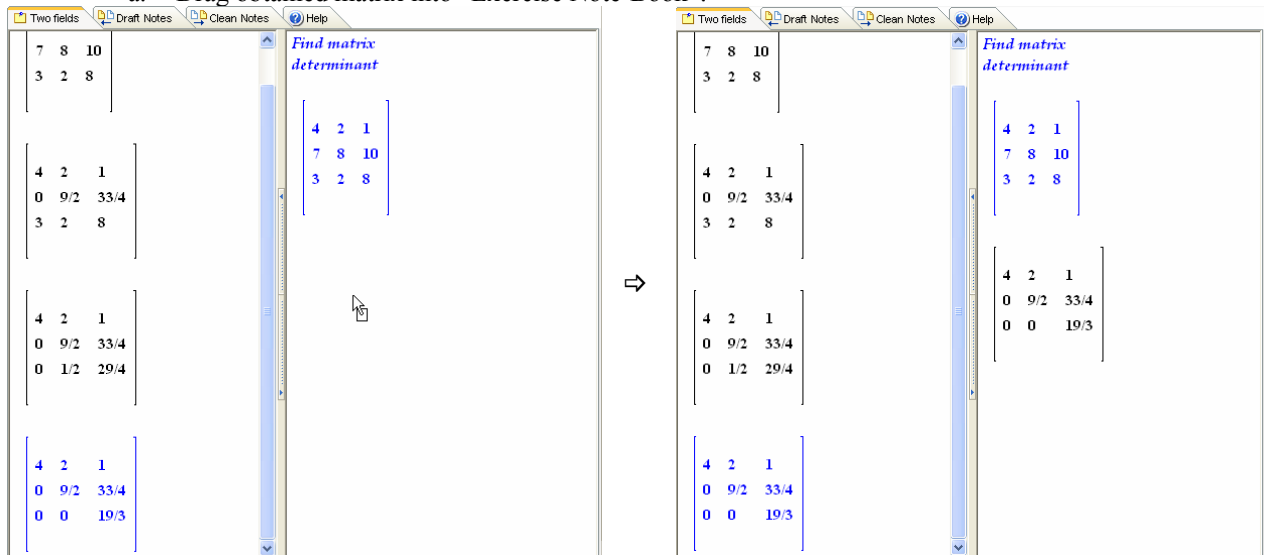
c. Drag the first element of the first matrix line to the first element of the third line of this matrix:

d. Drag the second element of the second matrix line to the second element of the third line of this matrix:

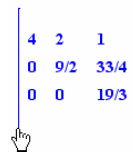


3. Assign the first control point:

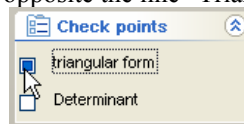
a. Drag obtained matrix into “Exercise Note-Book”:



b. Select dragged matrix by clicking on the left bracket:



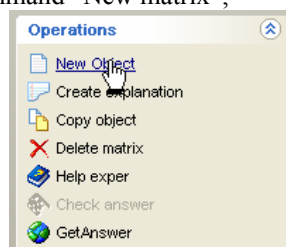
c. In menu “Control points” put a flag opposite the line “Triangular form”:



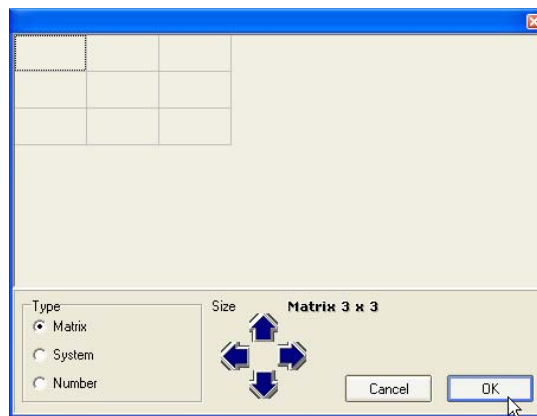
4. Multiply diagonal elements. You obtain determinant of this matrix.

5. Assign the second control point:

a. in menu “Operations” choose command “New matrix”;



b. in the box “New objecty ” assign “Type” as “Number” and input result:



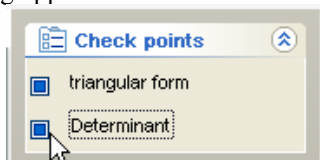
c. Drag this number to “Exercise Note-Book”.



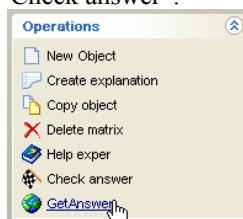
d. Select the number:



e. In menu “Control points” put flag opposite the line “Matrix determinant”



6. in menu “Operations” choose command “Check answer”:



7. If the task is solved correctly the lines of control points will light up green.

Practical tasks

Matrix determinant

Variant 1.

Find the matrix determinant

$$1. (\text{№}26) \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

$$2. (\text{№}33) \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$3. (\text{№}30) \begin{bmatrix} 3 & 3 & 0 & 2 \\ 1 & 0 & 3 & 2 \\ 3 & 2 & 1 & 2 \\ 2 & 3 & 2 & 1 \end{bmatrix}$$

$$4. (\text{№}41) \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$5. (\text{№}182) \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$$

Variant 2.

Find the matrix determinant

$$1. (\text{№}30) \begin{bmatrix} 3 & 3 & 0 & 2 \\ 1 & 0 & 3 & 2 \\ 3 & 2 & 1 & 2 \\ 2 & 3 & 2 & 1 \end{bmatrix}$$

$$2. (\text{№}36) \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 0 & 1 & 2 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

$$3. (\text{№}182) \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$$

$$4. (\text{№}38) \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$$

$$5. (\text{№}182) \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$$

Variant 3.

Find the matrix determinant

1. (№26)
$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

2. (№33)
$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

3. (№38)
$$\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$$

4. (№41)
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

5. (№182)
$$\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$$

Variant 4.

Find the matrix determinant

1. (№30)
$$\begin{bmatrix} 3 & 3 & 0 & 2 \\ 1 & 0 & 3 & 2 \\ 3 & 2 & 1 & 2 \\ 2 & 3 & 2 & 1 \end{bmatrix}$$

2. (№36)
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 0 & 1 & 2 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

3. (№26)
$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

4. (№41)
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

5. (№182)
$$\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$$

Variant 5.

Find the matrix determinant

1. (№26)
$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

2. (№33)
$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

3. (№182)
$$\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$$

4. (№38)
$$\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$$

5. (№41)
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$