

Laboratory Work №2

Reciprocation.

Background knowledge.

Elementary matrix. Nonsingular matrix. Inverse matrices. Method of Gaussian reciprocation.

Short theoretical information.

Definition. Square matrix A is a matrix, which has the inverse one, if there is a square matrix B , such that $AB=BA=E$.

Theorem. Square n matrix A has the inverse one if and only if matrix A is a nonsingular.

Manual «Linear algebra» O.V.Spivakovsky, V.A.Kreknin, pp. №№41-63 – Matrices and operations, pp. №№73-106 – determinants.

Solution algorithm.

For the solution of this task we use the property, according to which elementary transformations, modified nonsingular matrix to the identity one, can also modify identity matrix to such one which is inverse to it. Having compiled an augmented matrix, which includes parent matrix from the left and the identity one from the right, we can modify it. When in the left part we compile identity matrix, the matrix inverse of parent one will be in the right part.

Task. Compile inverse matrix.

$$(\text{№46}) \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix}.$$

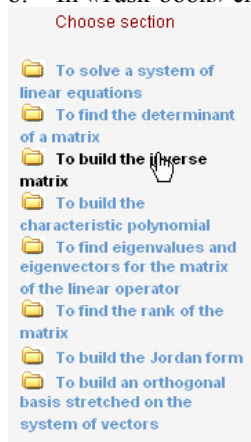
Algorithm of task download

1. Do next steps for task's download in the "Exercise-book":

- a. In the main menu choose "Task-book":



- b. In «Task-book» choose the chapter «Compile inverse matrix»:

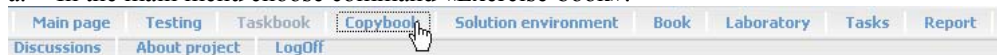


- c. Find task №46 and add it to the exercise-book



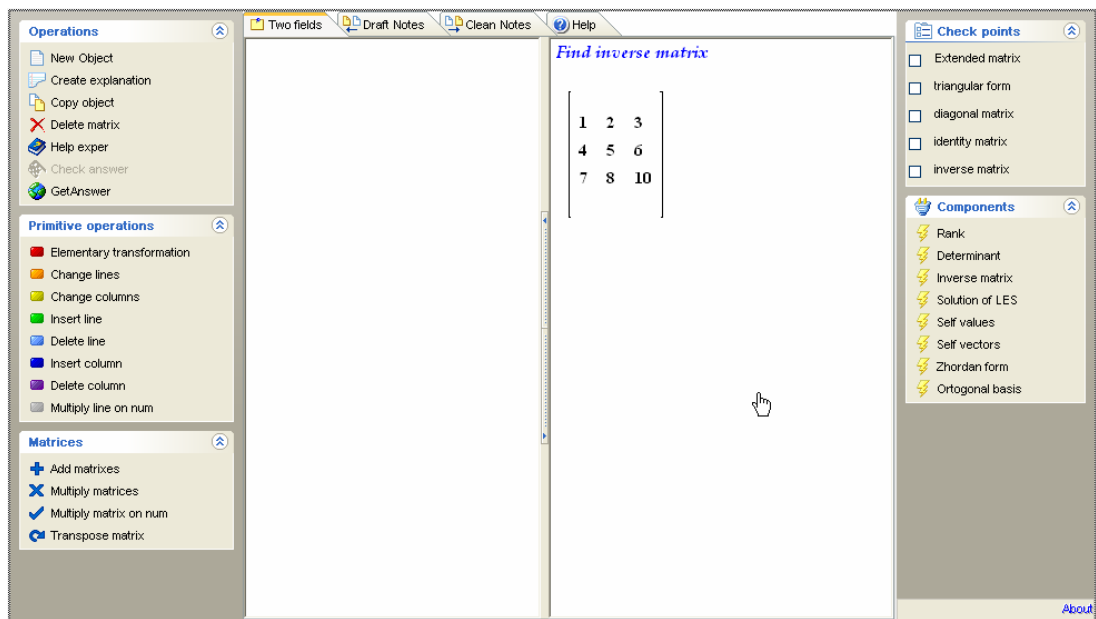
2. Move to exercise-book:

- a. In the main menu choose command «Exercise-book»:



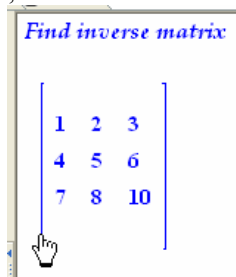
3. Find downloaded task and choose command "Solve"

After performing of all pointed steps «Solution environment» with the chosen task should be downloaded.

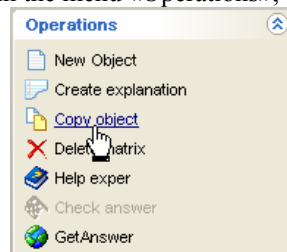


Solution algorithm

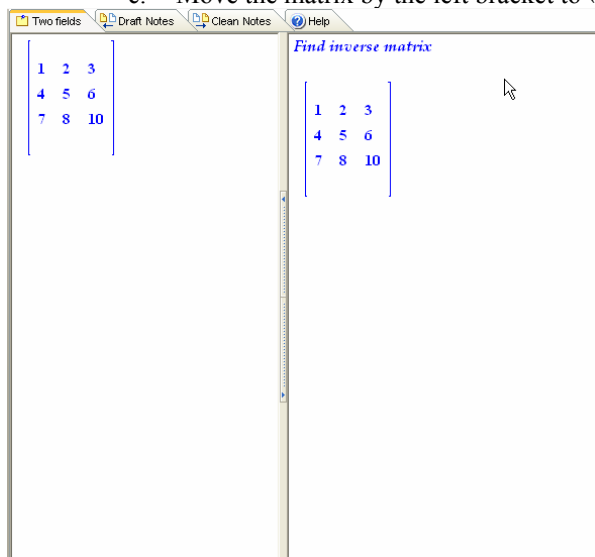
1. Copy matrix to a Draft:
 - a. Mark the matrix, pushing left bracket;



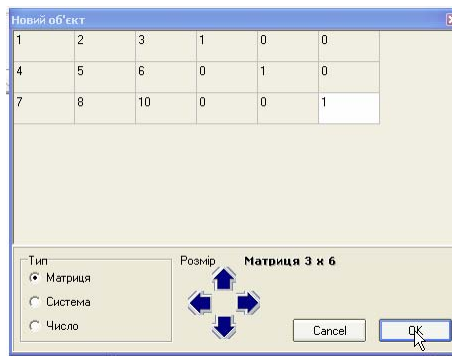
- b. Choose command "Copy object" in the menu «Operations»;



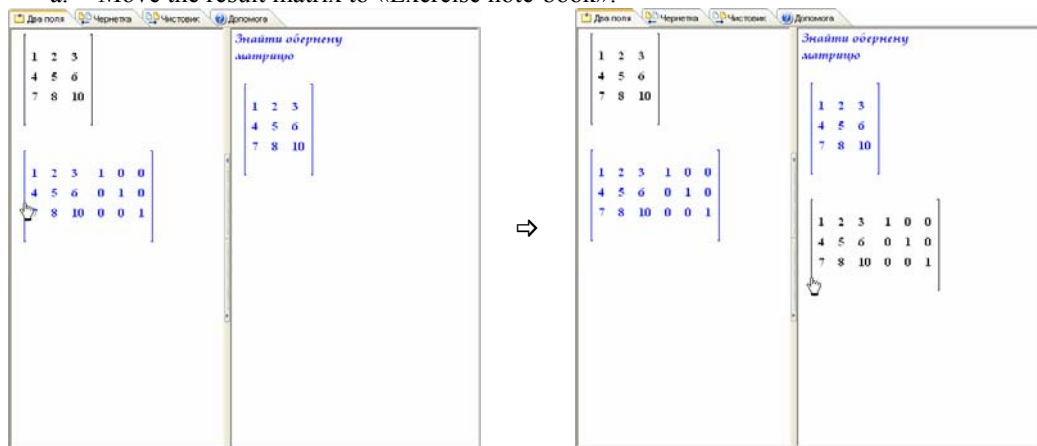
- c. Move the matrix by the left bracket to «Draft»:



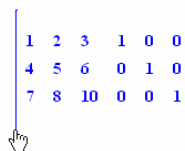
1. Compile augmented matrix.
 - a. In the menu «Operations» choose the command «New matrix»;



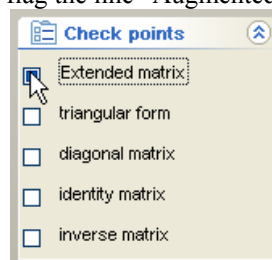
- i. Choose «Type of Matrix»;
 - ii. Point size, pushing the arrow;
 - iii. Print matrix elements.
- b. We compile augmented matrix
2. Set the first control point:
 - a. Move the result matrix to «Exercise note-book»:



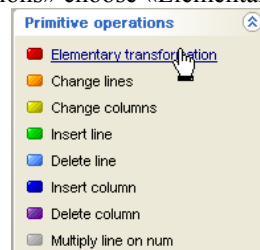
- b. Mark the moved matrix, pushing the left bracket



- c. In menu “Control point” mark by flag the line “Augmented matrix”:



3. Modify the left part of matrix to the triangle view.
 - a. In menu «Elementary transformations» choose «Elementary transformation»;



- b. Place the first element of the first matrix line to the first element of the second matrix line. System chooses the multiplier for the second line by itself and subtracts the second line from the first one;

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{bmatrix}$$
 $\Rightarrow$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{bmatrix}$$
 $\Rightarrow$

Operations

- New Object
- Create explanation
- Copy object
- Delete matrix
- Help exper
- Check answer
- GetAnswer

Primitive operations

- Elementary transformation
- Change lines
- Change columns
- Insert line
- Delete line
- Insert column
- Delete column
- Multiply line on num

Matrices

- Add matrixes
- Multiply matrices
- Multiply matrix on num
- Transpose matrix

Two fields Draft Notes Clean Notes Help

4 5 6
7 8 10

$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 4 & 6 & 2 & 0 & 0 \\ 8 & 10 & 12 & 0 & 2 & 0 \\ 14 & 16 & 20 & 0 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{bmatrix}$

Find inverse matrix

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{bmatrix}$

Check points

- ☒ Extended matrix
- ☐ triangular form
- ☐ diagonal matrix
- ☐ identity matrix
- ☐ inverse matrix

Components

- Rank
- Determinant
- Inverse matrix
- Solution of LES
- Self values
- Self vectors
- Zhordan form
- Ortogonal basis

About

c. Move the first element of the first matrix line to the first element of the third matrix line

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{bmatrix}$$
 $\Rightarrow$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -11 & -7 & 0 & 1 \end{bmatrix}$$
 $\Rightarrow$

Operations

- New Object
- Create explanation
- Copy object
- Delete matrix
- Help exper
- Check answer
- GetAnswer

Primitive operations

- Elementary transformation
- Change lines
- Change columns
- Insert line
- Delete line
- Insert column
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- Multiply line on num

Matrices

- Add matrixes
- Multiply matrices
- Multiply matrix on num
- Transpose matrix

Two fields Draft Notes Clean Notes Help

4 5 6 0 1 0
7 8 10 0 0 1

$\begin{bmatrix} 2 & 4 & 6 & 2 & 0 & 0 \\ 8 & 10 & 12 & 0 & 2 & 0 \\ 14 & 16 & 20 & 0 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -11 & -7 & 0 & 1 \end{bmatrix}$

Find inverse matrix

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{bmatrix}$

Check points

- ☒ Extended matrix
- ☐ triangular form
- ☐ diagonal matrix
- ☐ identity matrix
- ☐ inverse matrix

Components

- Rank
- Determinant
- Inverse matrix
- Solution of LES
- Self values
- Self vectors
- Zhordan form
- Ortogonal basis

About

d. Move the second element of the second matrix line to the second element of the third matrix line

The screenshot shows the software interface with the following components:

- Operations menu:** New Object, Create explanation, Copy object, Delete matrix, Help exper, Check answer, GetAnswer.
- Primitive operations menu:** Elementary transformation, Change lines, Change columns, Insert line, Delete line, Insert column, Delete column, Multiply line on num.
- Matrices menu:** Add matrixes, Multiply matrices, Multiply matrix on num, Transpose matrix.
- Check points sidebar:** Extended matrix, ☐ triangular form, ☐ diagonal matrix, ☐ identity matrix, ☐ inverse matrix.
- Components sidebar:** Rank, Determinant, Inverse matrix, Solution of LES, Self values, Self vectors, Zhordan form, Ortoogonal basis.

The main workspace displays several matrices, including the initial matrix $\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -11 & -7 & 0 & 1 \end{pmatrix}$ and the augmented matrix $\begin{pmatrix} 8 & 10 & 12 & 0 & 2 & 0 \\ 14 & 16 & 20 & 0 & 0 & 2 \end{pmatrix}$.

4. Set the second control point.

a. Move the result matrix to «Exercise note-book»:

The screenshot shows the software interface with the matrix $\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{pmatrix}$ being moved from the workspace to the 'Exercise note-book'.

b. Mark the moved matrix, pushing on the left bracket

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{pmatrix}$$

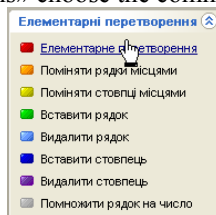
c. In menu«Control points» mark by flag the line «Triangle view»:

The screenshot shows the 'Control points' menu with the following options:

- ☒ Розширена матриця
- ☒ Трикутний вигляд
- ☐ Діагональна матриця
- ☐ Одичина матриця
- ☐ Обернена матриця

5. Modify matrix to diagonal view:

- a. In menu «Elementary transformations» choose the command “Elementary transformation»;



- b. The second element of the second line move to the second element of the first line;

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

⇒

$$\begin{bmatrix} 1 & 0 & -1 & (-5)/3 & 2/3 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

⇒

The screenshot shows the software interface with the following components:

- Left Sidebar (Операції):**
 - Нова матриця
 - Створити запис
 - Копіювати об'єкт
 - Видалити матрицю
 - Допомога експерта
 - Перевірити відповідь
 - GetAnswer
- Elementary transformations menu (Елементарні перетворення):**
 - Елементарне перетворення (highlighted)
 - Поміняти рядки місцями
 - Поміняти стовпці місцями
 - Вставити рядок
 - Видалити рядок
 - Вставити стовпець
 - Видалити стовпець
 - Помножити рядок на число
- Matrix section (Матриці):**
 - Додати матрицю
 - Помножити матрицю
 - Помножити матрицю на число
 - Транспонувати матрицю
- Main Window:**
 - Top tabs: Два поля, Чернетка, Чистовик, Допомога.
 - Left pane: Shows the initial matrix and the matrix after the first transformation.
 - Right pane: Shows the matrix after the second transformation.
 - Center pane: Shows the matrix after the third transformation.
- Right Sidebar (Контрольні точки):**
 - Розширена матриця
 - Трикутний вигляд
 - Діагональна матриця
 - Одинична матриця
 - Обернена матриця
- Components section (Компоненти):**
 - Ранг матриці
 - Визначник матриці
 - Обернена матриця
 - Розв'язок системи
 - Власні значення
 - Власні вектори
 - Жорданова форма
 - Ортогональний базис

- c. The third element of the third line move to the third element of the first line;

$$\begin{bmatrix} 1 & 0 & -1 & (-5)/3 & 2/3 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

⇒

$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

⇒

The screenshot shows the software interface with the following components:

- Left Sidebar (Операції):**
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 - Помножити матрицю
 - Помножити матрицю на число
 - Транспонувати матрицю
- Main Window:**
 - Top tabs: Два поля, Чернетка, Чистовик, Допомога.
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 - Розширена матриця
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 - Діагональна матриця
 - Одинична матриця
 - Обернена матриця
- Components section (Компоненти):**
 - Ранг матриці
 - Визначник матриці
 - Обернена матриця
 - Розв'язок системи
 - Власні значення
 - Власні вектори
 - Жорданова форма
 - Ортогональний базис

- d. The third element of third line move to the third element of the second line;

$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & -3 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$
 $\Rightarrow$

$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & -3 & 0 & 2 & -11 & 6 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$
 $\Rightarrow$

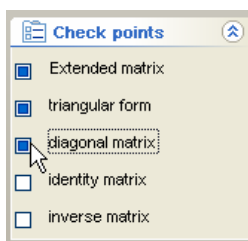
6. Set the first control point

- a. Move the result matrix to «Exercise note-book»

- b. Mark the result matrix, pushing the left bracket

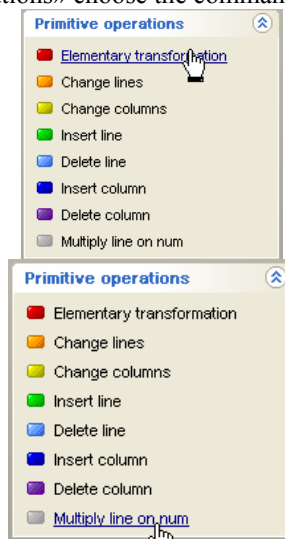
$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & -3 & 0 & 2 & -11 & 6 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

- c. In menu «Control points» mark by flag the line «Diagonal matrix»



7. Find matrix:

- a. In menu «Elementary transformations» choose the command “Elementary transformation»;



- b. Push non-identity element in the main matrix diagonal, find multiplier to it in a such way, that after multiplying we'll have 1, and push «↔»:

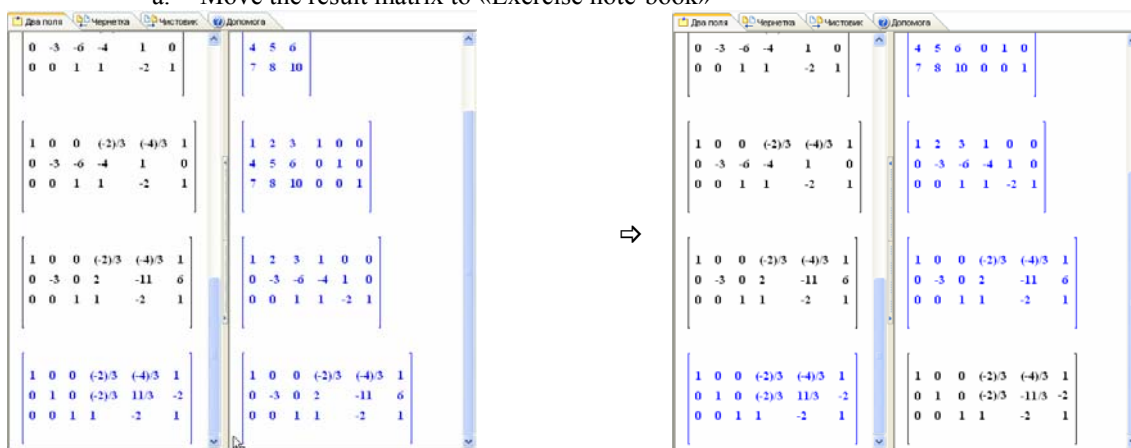
$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & -3 & 0 & 2 & -11 & 6 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

⇒

$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & -3 & 0 & 2 & -11 & 6 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

8. Set the next control point

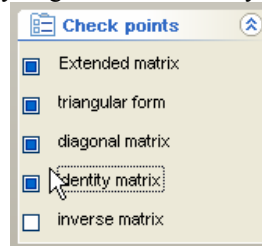
- a. Move the result matrix to «Exercise note-book»



- b. Mark the result matrix, pushing the left bracket

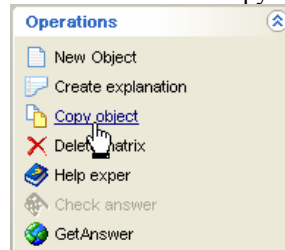
$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & 1 & 0 & (-2)/3 & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

- c. In menu «Control points» mark by flag the line «Identity matrix»



9. Copy the right part of augmented matrix

- a. In menu «Operations» choose the command «Copy object» ;



- b. Mark the last three matrix lines;

$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & 1 & 0 & (-2)/3 & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

- c. Move it down by the left bracket:

$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & 1 & 0 & (-2)/3 & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

⇒

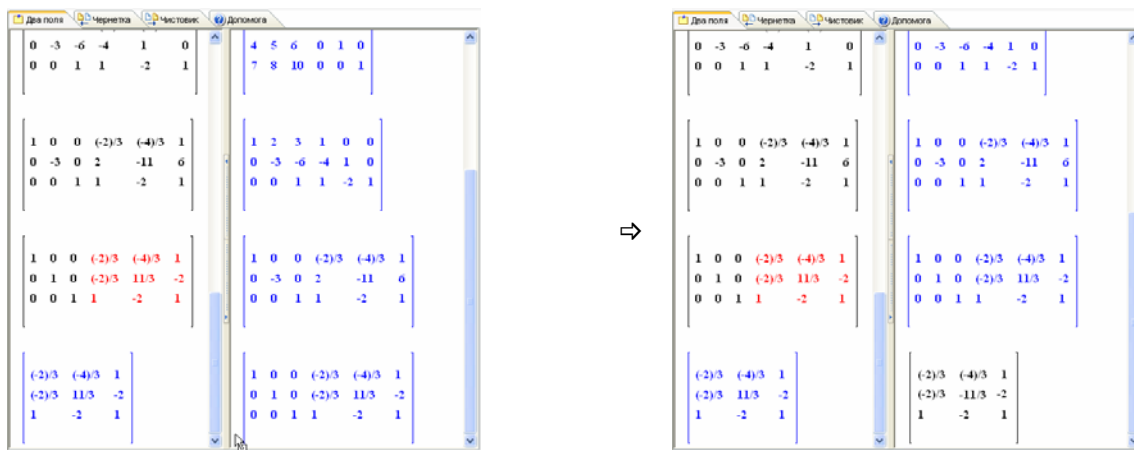
$$\begin{bmatrix} 1 & 0 & 0 & (-2)/3 & (-4)/3 & 1 \\ 0 & 1 & 0 & (-2)/3 & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} (-2)/3 & (-4)/3 & 1 \\ (-2)/3 & 11/3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

- d. This matrix is inverse matrix.

10. Set the last control point:

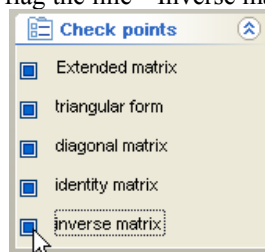
- a. Move the result matrix to «Exercise note-book»



- b. Mark the result matrix, pushing left bracket

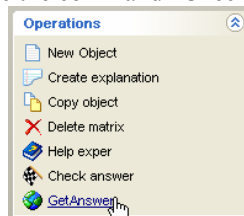
$$\begin{bmatrix} (-2)/3 & (-4)/3 & 1 \\ (-2)/3 & 11/3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

- c. In menu «Control point» mark by flag the line “Inverse matrix”



11. Check the correctness of your work:

- a. In menu «Operations», choose the command “Check answer”:



- b. If the task is correct, the lines of control points will become green

If you are mistaken, all the lines will be red.

Practical tasks

Constructing of inverse matrix

Variant 1.

Construct the inverse matrix

1. (№43) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2. . (№47) $\begin{bmatrix} 11 & 2 & 9 \\ 5 & 7 & 3 \\ 6 & 7 & 11 \end{bmatrix}$

3. (№50) $\begin{bmatrix} 5 & -2 & 0 & -1 \\ -2 & 1 & 1 & 1 \\ 4 & -3 & 5 & 2 \\ 1 & -1 & 2 & 1 \end{bmatrix}$

$$4. (\text{№}74) \begin{bmatrix} -2 & 6 & 5 & -4 \\ -3 & 7 & 5 & -4 \\ 0 & 0 & 2 & 0 \\ -1 & 2 & 2 & 0 \end{bmatrix}$$

$$5. (\text{№}75) \begin{bmatrix} 1 & -1 & -\frac{1}{2} & 1 \\ 3 & 0 & 0 & 1 \\ -2 & -1 & \frac{3}{2} & 1 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{4} & \frac{3}{2} \end{bmatrix}$$

Variant 2.

Construct the inverse matrix

$$1. (\text{№}83) \begin{bmatrix} 4 & -6 & 1 \\ 2 & -3 & \frac{1}{2} \\ 3 & -8 & 2 \\ 4 & -8 & 2 \end{bmatrix}$$

$$2. (\text{№}77) \begin{bmatrix} 3 & 4 & 4 & -14 \\ 2 & 2 & 0 & -4 \\ 3 & 4 & 2 & -10 \\ 3 & 2 & 1 & -5 \end{bmatrix}$$

$$3. (\text{№}72) \begin{bmatrix} 2 & -\frac{3}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 1 & 1 \\ 0 & -\frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$$

$$4. (\text{№}71) \begin{bmatrix} -1 & -2 & 1 & 2 \\ 2 & 3 & -\frac{3}{2} & -1 \\ 2 & 1 & \frac{1}{2} & -1 \\ -1 & -\frac{1}{2} & -\frac{1}{4} & \frac{5}{2} \end{bmatrix}$$

$$5. (\text{№}70) \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & -\frac{1}{3} & \frac{2}{3} \\ -1 & 0 & 2 & 0 \\ -1 & 0 & 0 & 2 \end{bmatrix}$$

Variant 3.

Construct the inverse matrix

$$1. (\text{№}84) \begin{bmatrix} 2 & 2 & -2 \\ 1 & \frac{1}{2} & -1 \\ 3 & 2 & -1 \end{bmatrix}$$

$$2. (\text{№}48) \begin{bmatrix} 2 & 3 & 4 & 5 \\ 13 & 11 & 7 & 6 \\ 17 & 19 & 1 & 4 \\ 5 & 7 & 11 & 2 \end{bmatrix}$$

$$3. (\text{№}50) \begin{bmatrix} 5 & -2 & 0 & -1 \\ -2 & 1 & 1 & 1 \\ 4 & -3 & 5 & 2 \\ 1 & -1 & 2 & 1 \end{bmatrix}$$

$$4. (\text{№}51) \begin{bmatrix} 1 & -1 & 2 & -1 \\ -1 & 2 & -3 & 2 \\ 2 & -3 & 6 & -1 \\ 1 & -2 & 5 & 3 \end{bmatrix}$$

$$5. (\text{№}52) \begin{bmatrix} 2 & 1 & 0 & -1 \\ 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & -1 \\ 1 & -1 & 2 & 1 \end{bmatrix}$$

Variant 4.

Construct the inverse matrix

$$1. (\text{№}43) \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$2. (\text{№}82) \begin{bmatrix} 3 & -\frac{1}{2} & -1 \\ 2 & \frac{5}{2} & -3 \\ \frac{1}{2} & \frac{5}{4} & -\frac{3}{2} \end{bmatrix}$$

$$3. (\text{№}53) \begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & -1 & 2 & 2 \\ -1 & -4 & 8 & 2 \\ 2 & 5 & -9 & 2 \end{bmatrix}$$

$$4. (\text{№}55) \begin{bmatrix} 4 & 1 & 2 & -1 \\ 4 & 4 & 3 & -1 \\ 3 & 3 & 2 & -1 \\ 2 & -1 & -1 & 1 \end{bmatrix}$$

$$5. (\text{№}56) \begin{bmatrix} -2 & 1 & 3 & -1 \\ 0 & 4 & 4 & -1 \\ 1 & 3 & 3 & -1 \\ 0 & -1 & -2 & 1 \end{bmatrix}$$

Variant 5.

Construct the inverse matrix

$$1. (\text{№}81) \begin{bmatrix} 3 & 0 & \frac{1}{2} \\ 1 & -1 & \frac{3}{2} \\ 2 & -1 & 1 \end{bmatrix}$$

$$2. (\text{№}73) \begin{bmatrix} \frac{1}{2} & -3 & -1 & 4 \\ -\frac{3}{4} & \frac{1}{2} & -\frac{1}{2} & 2 \\ -\frac{1}{2} & -1 & 1 & 2 \\ -1 & -2 & -1 & 5 \end{bmatrix}$$

$$3. (\text{№}57) \begin{bmatrix} -1 & -1 & -1 & 1 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 1 & -2 \\ 1 & 2 & -2 & -2 \end{bmatrix}$$

$$4. (\text{№}58) \begin{bmatrix} 2 & -3 & 4 & 3 \\ 0 & 2 & -2 & -1 \\ 0 & -4 & 3 & 2 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

$$5. (\text{№}59) \begin{bmatrix} -1 & -2 & 1 & -3 \\ 1 & 1 & 0 & 2 \\ 2 & 2 & 1 & 2 \\ 1 & 3 & -3 & 7 \end{bmatrix}$$

Variant 6.

Construct the inverse matrix

$$1. (\text{№}82) \begin{bmatrix} 3 & -\frac{1}{2} & -1 \\ 2 & \frac{5}{2} & -3 \\ \frac{1}{2} & \frac{5}{4} & -\frac{3}{2} \end{bmatrix}$$

$$2. (\text{№}60) \begin{bmatrix} 1 & 1 & -2 & -1 \\ 4 & -4 & -1 & 1 \\ -3 & 2 & 0 & -1 \\ -5 & -1 & 1 & 1 \end{bmatrix}$$

$$3. (\text{№}61) \begin{bmatrix} 2 & 3 & 2 & 2 \\ 3 & 2 & -3 & -1 \\ -2 & 0 & 3 & 2 \\ -2 & -1 & 2 & 1 \end{bmatrix}$$

$$4. (\text{№}62) \begin{bmatrix} -6 & -5 & 6 & 2 \\ 3 & 2 & -3 & -1 \\ -2 & 0 & 3 & 2 \\ -2 & -1 & 2 & 1 \end{bmatrix}$$

$$5. (\text{№}63) \begin{bmatrix} 1 & -1 & 2 & -2 \\ 0 & -1 & 2 & -1 \\ -1 & -1 & 3 & 2 \\ 2 & -1 & 2 & -4 \end{bmatrix}$$

Variant 7.

Construct the inverse matrix

$$1. (\text{№}84) \begin{bmatrix} 2 & 2 & -2 \\ 1 & \frac{1}{2} & -1 \\ 3 & 2 & -1 \end{bmatrix}$$

$$2. (\text{№}64) \begin{bmatrix} -4 & -2 & -3 & 2 \\ 2 & 0 & 1 & -1 \\ 3 & 3 & 2 & -1 \\ -2 & -1 & -1 & 1 \end{bmatrix}$$

$$3. (\text{№}65) \begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & -1 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 2 & -3 & 1 & -1 \end{bmatrix}$$

$$4. (\text{№}66) \begin{bmatrix} 3 & -1 & -\frac{3}{2} & 1 \\ 3 & 0 & -2 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & -\frac{3}{2} & 3 \end{bmatrix}$$

$$5. (\text{№}50) \begin{bmatrix} 5 & -2 & 0 & -1 \\ -2 & 1 & 1 & 1 \\ 4 & -3 & 5 & 2 \\ 1 & -1 & 2 & 1 \end{bmatrix}$$

Variant 8.

Construct the inverse matrix

$$1. (\text{№}43) \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$2. (\text{№}85) \begin{bmatrix} 2 & \frac{3}{2} & -\frac{1}{2} \\ 1 & \frac{3}{2} & -\frac{1}{2} \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned}
3. (\text{№}50) & \begin{bmatrix} 5 & -2 & 0 & -1 \\ -2 & 1 & 1 & 1 \\ 4 & -3 & 5 & 2 \\ 1 & -1 & 2 & 1 \end{bmatrix} \\
4. (\text{№}68) & \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} & \frac{3}{4} & 0 \\ -2 & 5 & 2 & -4 \\ -1 & 1 & \frac{3}{2} & 0 \\ -\frac{5}{2} & \frac{9}{2} & \frac{7}{4} & -3 \end{bmatrix} \\
5. (\text{№}69) & \begin{bmatrix} 3 & 1 & -1 & -1 \\ -2 & 3 & 1 & 1 \\ \frac{5}{2} & \frac{1}{2} & 0 & -\frac{3}{2} \\ \frac{2}{2} & \frac{2}{2} & 0 & \frac{2}{2} \\ 0 & 1 & 0 & 1 \end{bmatrix}
\end{aligned}$$

Variant 9.

Construct the inverse matrix

$$\begin{aligned}
1. (\text{№}47) & \begin{bmatrix} 11 & 2 & 9 \\ 5 & 7 & 3 \\ 6 & 7 & 11 \end{bmatrix} \\
2. (\text{№}78) & \begin{bmatrix} 2 & -1 & \frac{3}{2} \\ 1 & -1 & 0 \\ -4 & 4 & 2 \end{bmatrix} \\
3. (\text{№}73) & \begin{bmatrix} \frac{1}{2} & -3 & -1 & 4 \\ -\frac{3}{4} & \frac{1}{2} & -\frac{1}{2} & 2 \\ -\frac{1}{2} & -1 & 1 & 2 \\ -1 & -2 & -1 & 5 \end{bmatrix} \\
4. (\text{№}70) & \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & -\frac{1}{3} & \frac{2}{3} \\ -1 & 0 & 2 & 0 \\ -1 & 0 & 0 & 2 \end{bmatrix} \\
5. (\text{№}50) & \begin{bmatrix} 5 & -2 & 0 & -1 \\ -2 & 1 & 1 & 1 \\ 4 & -3 & 5 & 2 \\ 1 & -1 & 2 & 1 \end{bmatrix}
\end{aligned}$$

Variant 10.

Construct the inverse matrix

$$1. (\text{№}86) \begin{bmatrix} 4 & \frac{5}{2} & -4 \\ 2 & 4 & -4 \\ 3 & \frac{7}{2} & -4 \end{bmatrix}$$

$$2. (\text{№}79) \begin{bmatrix} -1 & 2 & 4 \\ -1 & 1 & 1 \\ \frac{2}{3} & 0 & 0 \end{bmatrix}$$

$$3. (\text{№}73) \begin{bmatrix} \frac{1}{2} & -3 & -1 & 4 \\ -\frac{3}{4} & \frac{1}{2} & -\frac{1}{2} & 2 \\ -\frac{1}{2} & -1 & 1 & 2 \\ -1 & -2 & -1 & 5 \end{bmatrix}$$

$$4. (\text{№}61) \begin{bmatrix} 2 & 3 & 2 & 2 \\ 3 & 2 & -3 & -1 \\ -2 & 0 & 3 & 2 \\ -2 & -1 & 2 & 1 \end{bmatrix}$$

$$5. (\text{№}65) \begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & -1 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 2 & -3 & 1 & -1 \end{bmatrix}$$

Questions for self-control:

1. Give the definition of inverse matrix?
2. What is the nonsingular matrix?
3. What is the sum of singular matrices?
4. How many inverse matrices can we compile for matrix A?
5. Are the lines of inverse matrix linearly dependent or not?
6. Matrix has zero line. Can we compile the inverse matrix for it?
7. What algorithm should we use for inverse 4x5 matrix compiling?
8. Has the elementary matrix an inverse one?