

Laboratory work №3

Systems of Linear Equations.

Background knowledge.

The definition of the system and solving systems of linear equations. Equal systems. Rank of basic and augmented matrix. Specified and fundamental solution of the system.

Brief theoretical information.

Definition. A system of linear equations is consistent if it has at least one solution. A system of linear equations is inconsistent if it has no solutions.

Theorem. To consider a system consistent the necessary and sufficient condition is that the rank of basic matrix of the system be equal to the rank of augmented matrix.

Any solution of heterogeneous system of linear equations may be obtained as the sum of any particular solution of the system and arbitrary solution of the corresponding homogeneous system. If the system is consistent, it is solved by means of exclusion of unknown variables – Gauss method or Cramer's Rule.

📖 O.V. Spivakovsky and V.A. Kreknin. Linear Algebra.

Algorithm of Solution.

To solve a system of linear equations applying Gauss method we need by means of elementary transformations reduce the basic matrix to diagonality, when the principal diagonal consists of one's (actually to identity matrix), to find the ranks of basic and augmented matrixes of the system, to determine consistency of the system, to determine the number of solutions. If the system has a unique solution, then the far right column of augmented matrix will be the column-vector of solutions. If there are infinitely many solutions, it is necessary to choose main and unrestricted variables, to make specified and fundamental solution.

When in the system of linear equations $m=n$ and augmented matrix A is nondegenerate, then the system has a unique solution, which is written in the form of Cramer's formulas: $x_i = |B_i|/|A|$, where B_i - matrix, obtained from A by means of replacing of coefficient column of variable x_i by the column of absolute terms.

Task. Solve the following system of linear equations

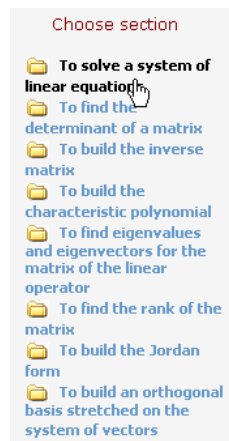
$$(№88) \begin{cases} 3 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 - 2 \cdot x_4 + 2 \cdot x_5 = 11 \\ 1 \cdot x_1 + 1 \cdot x_2 + 1 \cdot x_3 - 1 \cdot x_4 + 1 \cdot x_5 = 5 \\ 0 \cdot x_1 + 2 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4 + 1 \cdot x_5 = 4 \\ 4 \cdot x_1 - 3 \cdot x_2 + 0 \cdot x_3 - 6 \cdot x_4 + 5 \cdot x_5 = 20 \\ 3 \cdot x_1 + 1 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4 - 2 \cdot x_5 = 0 \end{cases}$$

Algorithm of loading of the task

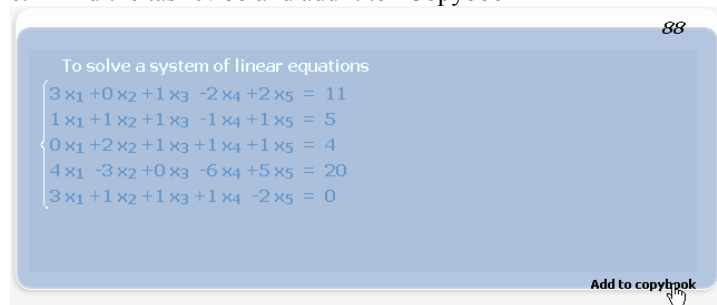
1. To download the task into "Exercise book" do the following:
 - a. Choose "Taskbook" in the main menu:



- b. In the "Taskbook" choose unit "Build up Matrix Inverse":



- c. find the task №88 and add it to “Copybook”

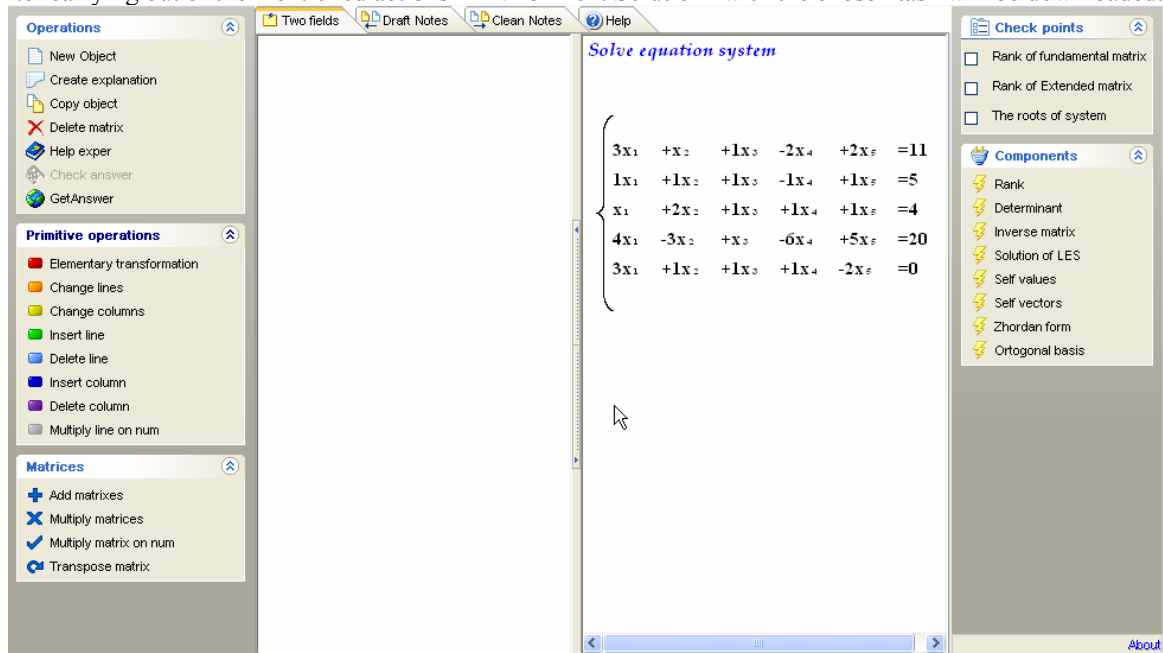


2. Go to Exercise book:
- a. Choose “Copybook” in the main menu:



3. Find the loaded task and choose operation “Solve”

After carrying out of the mentioned actions “Environment Solution” with the chosen task will be downloaded.



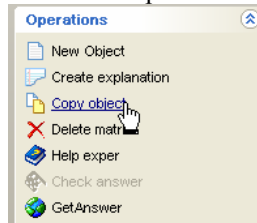
Algorithm of the task solution

1. Copy the matrix into the Rough Note- book:
- a. Select the matrix by clicking on the left arc;

Solve equation system

$$\begin{cases} 3x_1 + x_2 + 1x_3 - 2x_4 + 2x_5 = 11 \\ 1x_1 + 1x_2 + 1x_3 - 1x_4 + 1x_5 = 5 \\ x_1 + 2x_2 + 1x_3 + 1x_4 + 1x_5 = 4 \\ 4x_1 - 3x_2 + x_3 - 6x_4 + 5x_5 = 20 \\ 3x_1 + 1x_2 + 1x_3 + 1x_4 - 2x_5 = 0 \end{cases}$$

b. Choose operation “Copy Object” in the menu “Operations”;

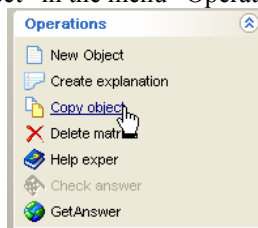


c. Drag the matrix into the “Rough Note-Book” holding down the left arc:

The software interface is shown with the 'Operations' menu on the left and the 'Rough Note-Book' in the center. The menu includes 'Operations' (with 'Copy object' highlighted), 'Primitive operations', and 'Matrices'. The 'Rough Note-Book' contains a matrix with five columns and five rows. The first four columns are highlighted in blue, indicating they are being copied. The right side of the interface shows a 'Check points' section with checkboxes for 'Rank of fundamental matrix', 'Rank of Extended matrix', and 'The roots of system', and a 'Components' section with various mathematical tools like 'Rank', 'Determinant', 'Inverse matrix', etc.

2. Copy the basic matrix

a. Choose operation “Copy Object” in the menu “Operations”;



b. Select the basic matrix (first four columns):

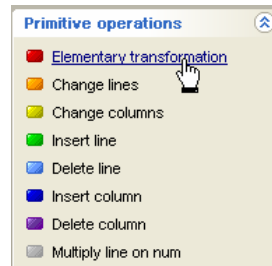
The diagram shows the matrix from the previous screenshot. The first four columns are highlighted in red, and a green arrow points to the bottom of the first column, indicating the selection process.

c. Drag the matrix down holding the left arc

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 & 11 \\ 1 & 1 & 1 & -1 & 1 & 5 \\ 0 & 2 & 1 & 1 & 1 & 4 \\ 4 & -3 & 0 & -6 & 5 & 20 \\ 3 & 1 & 1 & 1 & -2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 & 11 \\ 1 & 1 & 1 & -1 & 1 & 5 \\ 0 & 2 & 1 & 1 & 1 & 4 \\ 4 & -3 & 0 & -6 & 5 & 20 \\ 3 & 1 & 1 & 1 & -2 & 0 \end{bmatrix}$$

3. Find the rank of basic matrix:

a. Choose operation “Elementary Transformation” in the menu “Elementary Transformations”:



b. Drag the first element of the first matrix row to the first element of the second matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 1 & 1 & 1 & -1 & 1 \\ 0 & 2 & 1 & 1 & 1 \\ 4 & -3 & 0 & -6 & 5 \\ 3 & 1 & 1 & 1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 2 & 1 & 1 & 1 \\ 4 & -3 & 0 & -6 & 5 \\ 3 & 1 & 1 & 1 & -2 \end{bmatrix}$$

c. Drag the first element of the first matrix row to the first element of the fourth matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 2 & 1 & 1 & 1 \\ 4 & -3 & 0 & -6 & 5 \\ 3 & 1 & 1 & 1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & -3 & -4/3 & -10/3 & 7/3 \\ 3 & 1 & 1 & 1 & -2 \end{bmatrix}$$

d. Drag the first element of the first matrix row to the first element of the fifth matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & -3 & -4/3 & -10/3 & 7/3 \\ 3 & 1 & 1 & 1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & -3 & -4/3 & -10/3 & 7/3 \\ 0 & 1 & 0 & 3 & -4 \end{bmatrix}$$

e. Drag the second element of the second matrix row to the second element of the third matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & -3 & -4/3 & -10/3 & 7/3 \\ 0 & 1 & 0 & 3 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 \\ 0 & -3 & -4/3 & -10/3 & 7/3 \\ 0 & 1 & 0 & 3 & -4 \end{bmatrix}$$

f. Drag the second element of the second matrix row to the second element of the fourth matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 \\ 0 & -3 & -4/3 & -10/3 & 7/3 \\ 0 & 1 & 0 & 3 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 \\ 0 & 0 & 2/3 & -13/3 & 10/3 \\ 0 & 1 & 0 & 3 & -4 \end{bmatrix}$$

- g. Drag the second element of the second matrix row to the second element of the fifth row matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 \\ 0 & 0 & 2/3 & -13/3 & 10/3 \\ 0 & 1 & 0 & 3 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 \\ 0 & 0 & 2/3 & -13/3 & 10/3 \\ 0 & 0 & -2/3 & 10/3 & -13/3 \end{bmatrix}$$

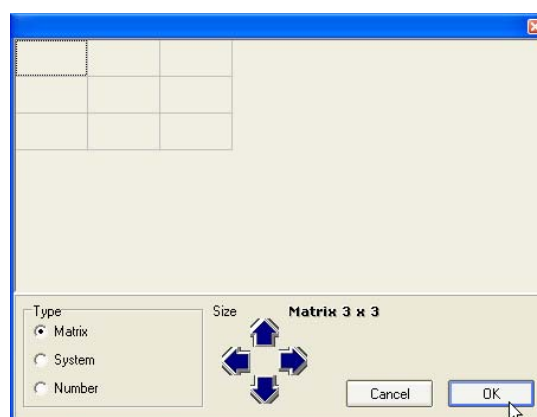
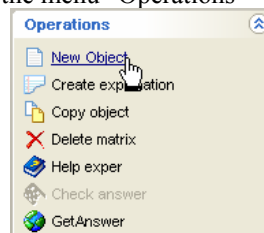
- h. Drag the third element of the third matrix row to the third element of the fourth matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 \\ 0 & 0 & 2/3 & -13/3 & 10/3 \\ 0 & 0 & -2/3 & 10/3 & -13/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & -2/3 & 10/3 & -13/3 \end{bmatrix}$$

- i. Drag the third element of the third matrix row to the third element of the fifth matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & -2/3 & 10/3 & -13/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 \\ 0 & 1 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix}$$

- j. All the elements under the main diagonal are equal to zero, so the matrix is modified to a triangle form;
 k. Rank of the basic matrix is 5, as the number of non-zero rows of a triangle matrix is 5;
 l. Choose operation “New object” in the menu “Operations”

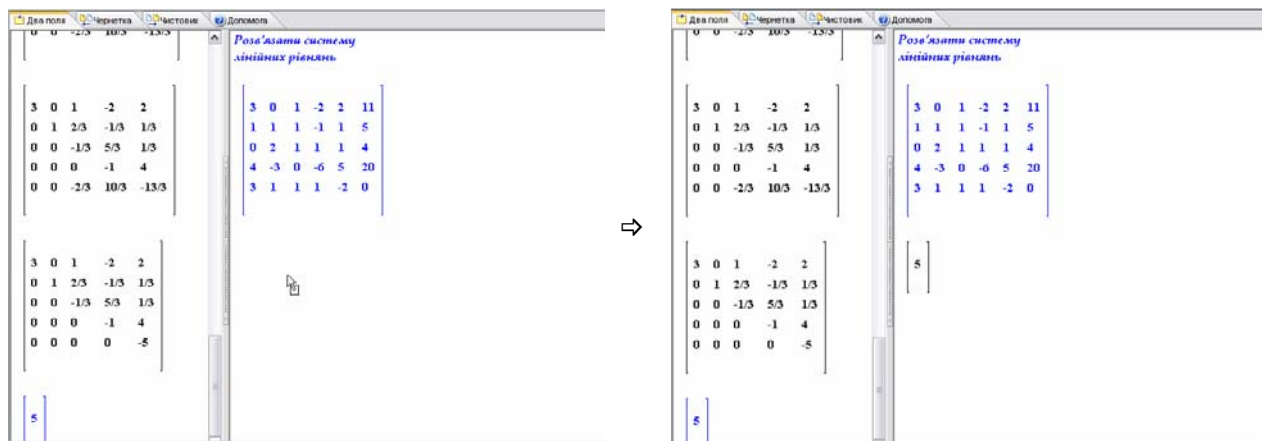


- m. In the window «A new object» establish the “Type” as “Number” and type number 5 in:

$$\begin{bmatrix} 5 \end{bmatrix}$$

4. Set the first program breakpoint:

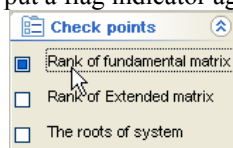
- a. Drag the obtained number to “Exercise Note-book”:



- b. Select the transported number clicking on the left arc:



- c. In the menu «Program Breakpoints» put a flag indicator against the row »Rank of the Basic Matrix»:



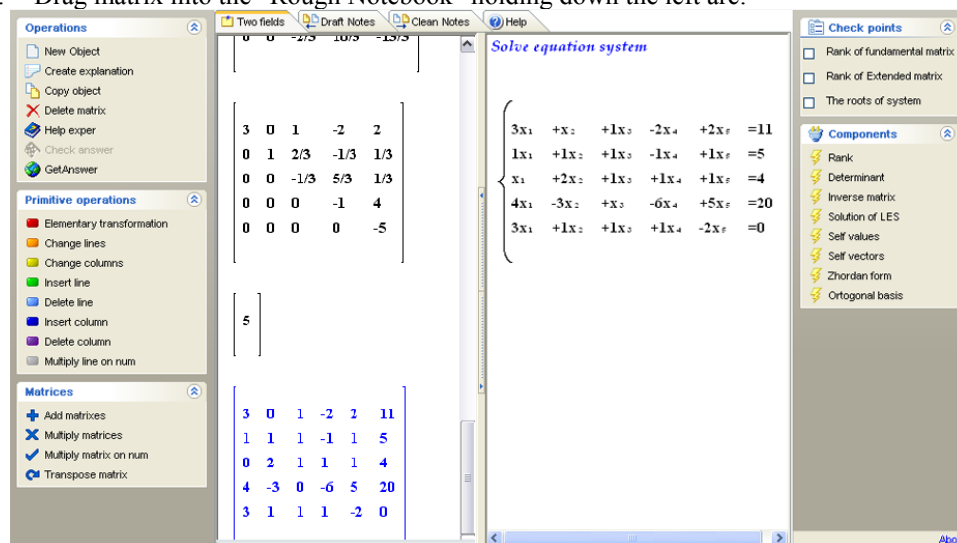
5. Copy the given matrix into the “Rough Note-book” one more time:

- a. Select matrix, clicking on the left arc;



- b. Choose operation “Copy Object” in the menu “Operations”;

- c. Drag matrix into the “Rough Notebook” holding down the left arc:



6. Find the rank of augmented matrix:

- a. Choose operation “Elementary Transformation” in the menu “Elementary Transformations” :



c. Drag the first element of the first matrix row to the first element of the fourth matrix row

d. Drag the first element of the first matrix row to the first element of the fifth matrix row

e. Drag the second element of the second matrix row to the second element of the third matrix row

f. Drag the second element of the second matrix row to the second element of the fourth matrix row

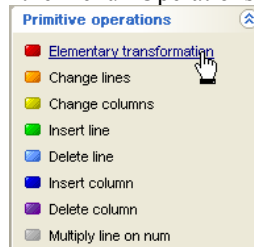
g. Drag the second element of the second matrix row to the second element of the fifth matrix row

h. Drag the third element of the third matrix row to the third element of the fourth matrix row

i. Drag the third element of the third matrix row to the third element of the fifth matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 & 11 \\ 0 & 1 & 2/3 & -1/3 & 1/3 & 4/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & -2/3 & 10/3 & -13/3 & -37/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 & 11 \\ 0 & 1 & 2/3 & -1/3 & 1/3 & 4/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- j. All the elements under the main diagonal are equal to zero, so the matrix is modified to a triangle form;
- k. Rank of the basic matrix is 5, as the number of non-zero rows of a triangle matrix is 5;
- l. Choose operation "A new matrix" in the menu "Operations"



- m. In the window «A new object» establish the "Type" as "Number" and type number 5 in:

5

7. Set the second program breakpoint:

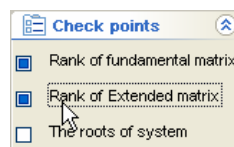
- a. Drag the obtained number to "Exercise Notebook":



- b. Select the transported number clicking the left arc:

5

- c. In the menu «Program Breakpoints» put a flag indicator against the row "Rank of the Augmented Matrix":

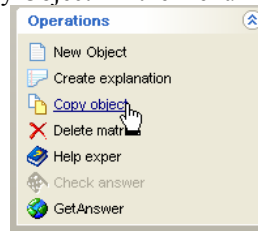


- d. Reduce the main part of augmented matrix to identity matrix:

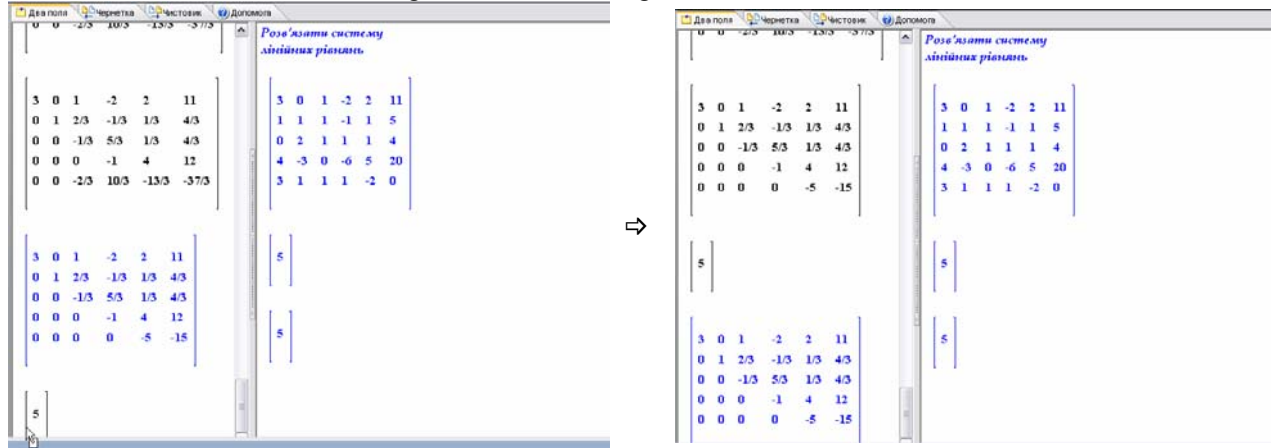
- a. Copy a triangle form of augmented matrix:

- i. select the matrix:

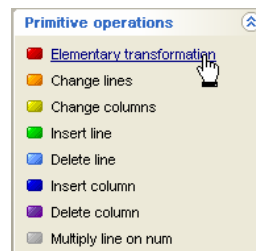
- ii. choose operation “Copy Object” in the menu “Operations”



- iii. Drag matrix down holding the left arc:



- b. Choose operation “Elementary Transformation” in the menu “Elementary Transformations”:



- c. Drag the third element of the third matrix row to the third element of the first matrix row

$$\begin{bmatrix} 3 & 0 & 1 & -2 & 2 & 11 \\ 0 & 1 & 2/3 & -1/3 & 1/3 & 4/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 & -2 & 2 & 11 \\ 0 & 1 & 2/3 & -1/3 & 1/3 & 4/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- d. Drag the fourth element of the fourth matrix row to the fourth element of the first matrix row:

$$\begin{bmatrix} 3 & 0 & 0 & 3 & 3 & 15 \\ 0 & 1 & 2/3 & -1/3 & 1/3 & 4/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 & 15 & 51 \\ 0 & 1 & 2/3 & -1/3 & 1/3 & 4/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- e. Drag the fifth element of the fifth matrix row to the fifth element of the first matrix row:

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 15 & 51 \\ 0 & 1 & 2/3 & -1/3 & 1/3 & 4/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 2/3 & -1/3 & 1/3 & 4/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- f. Drag the third element of the third matrix row to the third element of the second matrix row:

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 2/3 & -1/3 & 1/3 & 4/3 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 & 1 & 4 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- g. Drag the fourth element of the fourth matrix row to the fourth element of the second matrix row:

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 & 1 & 4 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 13 & 40 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- h. Drag the fifth element of the fifth matrix row to the fifth element of the second matrix row:

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 13 & 40 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- i. Drag the fourth element of the fourth matrix row to the fourth element of the third matrix row:

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1/3 & 5/3 & 1/3 & 4/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1/3 & 0 & 7 & 64/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- j. Drag the fifth element of the fifth matrix row to the fifth element of the third matrix row:

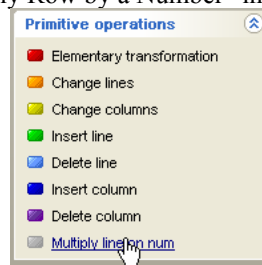
$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1/3 & 0 & 7 & 64/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1/3 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- k. Drag the fifth element of the fifth matrix row to the fifth element of the fourth matrix row:

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1/3 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & -1 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1/3 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- l. So the diagonal matrix was obtained;

- m. Choose operation “Multiply Row by a Number” in the menu “Operations”:



- n. Click on the first element of the first row, find a multiplying coefficient and press «↔»

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1/3 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- o. Click on the third element of the third row, find a multiplying coefficient and press «↔»

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- p. Click on the fourth element of the fourth row, find a multiplying coefficient and press «↔»

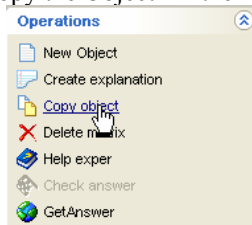
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & -15 \end{bmatrix}$$

- q. Click on the fifth element of the fifth row, find a multiplying coefficient and press «↔»

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

- e. The last column of augmented matrix contains the roots of the given system, copy it:

- a. Choose operation “Copy the Object” in the menu “Operations”;



- b. select the last column

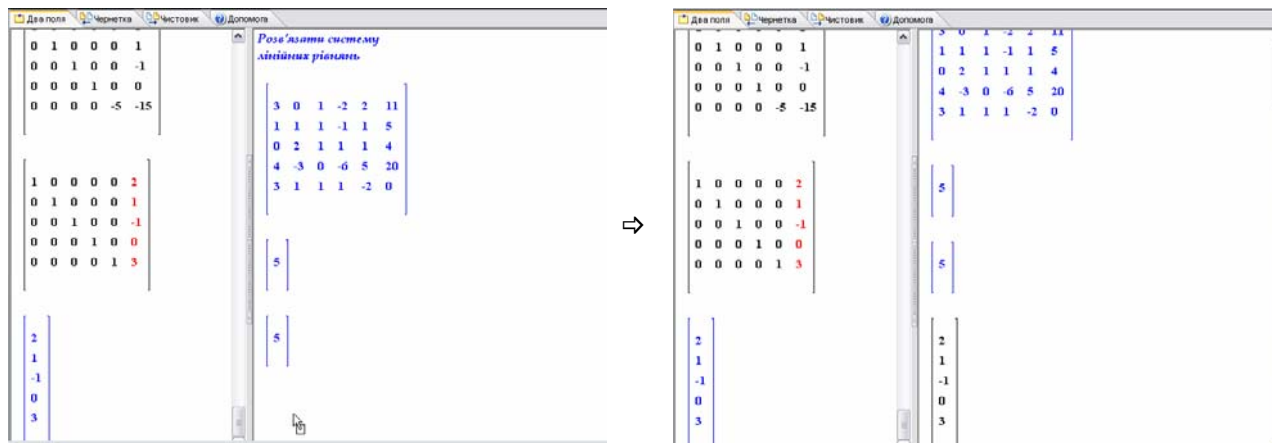
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

- c. drag the matrix down holding the left arc

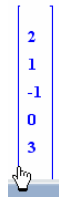
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

- f. Set the last program breakpoint:

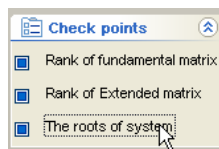
- a. Drag the obtained roots to “Exercise-Notebook”:



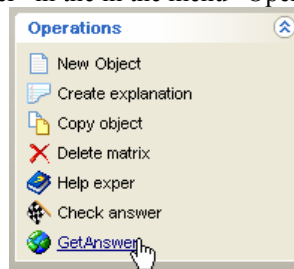
b. Select the transported roots clicking the left arc:



c. In the menu «Program Breakpoints» put a flag indicator against the row ”The roots of system”:



g. Choose operation “GetAnswer” in the in the menu “Operations”:



h. If the task was solved properly, the rows of program breakpoints will become green; otherwise the row with the fault program breakpoint will become red.

Practical tasks

Systems of simple equation

Variant 1.

Solve the systems of simple equations

$$\begin{aligned} 1. \text{ (№42)} & \begin{cases} 1 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 = 4 \\ 8 \cdot x_1 + 7 \cdot x_2 + 6 \cdot x_3 = 5 \\ 9 \cdot x_1 + 10 \cdot x_2 + 11 \cdot x_3 = 12 \end{cases} \\ 2. \text{ (№102)} & \begin{cases} 1 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 - 3 \cdot x_4 = 0 \\ 4 \cdot x_1 - 1 \cdot x_2 - 5 \cdot x_3 + 14 \cdot x_4 = 0 \\ 2 \cdot x_1 + 5 \cdot x_2 + 3 \cdot x_3 - 4 \cdot x_4 = 0 \\ 3 \cdot x_1 + 2 \cdot x_2 - 1 \cdot x_3 + 5 \cdot x_4 = 0 \end{cases} \\ 3. \text{ (№87)} & \begin{cases} 2 \cdot x_1 - 1 \cdot x_2 - 1 \cdot x_3 - 1 \cdot x_4 - 1 \cdot x_5 = 1 \\ 3 \cdot x_1 - 6 \cdot x_2 + 4 \cdot x_3 - 3 \cdot x_4 + 0 \cdot x_5 = 14 \\ 2 \cdot x_1 - 2 \cdot x_2 + 3 \cdot x_3 + 0 \cdot x_4 + 1 \cdot x_5 = 8 \\ 1 \cdot x_1 - 1 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4 + 1 \cdot x_5 = 2 \\ 1 \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + 2 \cdot x_4 + 1 \cdot x_5 = -1 \end{cases} \\ 4. \text{ (№98)} & \begin{cases} 1 \cdot x_1 + 1 \cdot x_2 + 2 \cdot x_3 + 3 \cdot x_4 + 3 \cdot x_5 = 3 \\ 2 \cdot x_1 + 3 \cdot x_2 - 1 \cdot x_3 - 2 \cdot x_4 - 7 \cdot x_5 = -1 \\ 2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 + 1 \cdot x_5 = 2 \\ 1 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 + 3 \cdot x_5 = 3 \\ 3 \cdot x_1 - 1 \cdot x_2 + 0 \cdot x_3 + 4 \cdot x_4 - 2 \cdot x_5 = 0 \end{cases} \\ 5. \text{ (№112)} & \begin{cases} 2 \cdot x_1 + 1 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 = 1 \\ 3 \cdot x_1 - 2 \cdot x_2 + 1 \cdot x_3 + 4 \cdot x_4 = -4 \\ 1 \cdot x_1 + 3 \cdot x_2 - 2 \cdot x_3 + 3 \cdot x_4 = 5 \\ 2 \cdot x_1 + 3 \cdot x_2 - 2 \cdot x_3 + 1 \cdot x_4 = 12 \end{cases} \end{aligned}$$

Variant 2.

Solve the systems of simple equations

$$\begin{aligned} 1. \text{ (№114)} & \begin{cases} 1 \cdot x_1 + 4 \cdot x_2 + 1 \cdot x_3 = 12 \\ -3 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 = 2 \\ 3 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 = 3 \end{cases} \\ 2. \text{ (№103)} & \begin{cases} 3 \cdot x_1 + 1 \cdot x_2 + 1 \cdot x_3 - 7 \cdot x_4 = 0 \\ 4 \cdot x_1 - 3 \cdot x_2 + 10 \cdot x_3 - 5 \cdot x_4 = 0 \\ 2 \cdot x_1 + 3 \cdot x_2 - 4 \cdot x_3 - 7 \cdot x_4 = 0 \\ 1 \cdot x_1 + 4 \cdot x_2 - 7 \cdot x_3 - 6 \cdot x_4 = 0 \end{cases} \\ 3. \text{ (№89)} & \begin{cases} 1 \cdot x_1 - 4 \cdot x_2 + 3 \cdot x_3 - 1 \cdot x_4 + 0 \cdot x_5 = -9 \\ 1 \cdot x_1 + 3 \cdot x_2 + 0 \cdot x_3 - 1 \cdot x_4 + 2 \cdot x_5 = 0 \\ 2 \cdot x_1 + 3 \cdot x_2 - 2 \cdot x_3 + 1 \cdot x_4 - 1 \cdot x_5 = 6 \\ 1 \cdot x_1 + 6 \cdot x_2 - 2 \cdot x_3 + 1 \cdot x_4 + 1 \cdot x_5 = 11 \\ 1 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 - 1 \cdot x_4 + 1 \cdot x_5 = -4 \\ 2 \cdot x_1 + 1 \cdot x_2 + 3 \cdot x_3 - 2 \cdot x_4 + 2 \cdot x_5 = -5 \end{cases} \end{aligned}$$

$$4. (\text{№}99) \begin{cases} 1 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 - 4 \cdot x_4 - 1 \cdot x_5 = -2 \\ 2 \cdot x_1 - 1 \cdot x_2 + 3 \cdot x_3 + 0 \cdot x_4 - 1 \cdot x_5 = -1 \\ 4 \cdot x_1 + 2 \cdot x_2 - 1 \cdot x_3 + 3 \cdot x_4 + 5 \cdot x_5 = 19 \\ 3 \cdot x_1 - 2 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 + 0 \cdot x_5 = 3 \\ 2 \cdot x_1 + 1 \cdot x_2 + 4 \cdot x_3 - 3 \cdot x_4 - 2 \cdot x_5 = -4 \end{cases}$$

$$5. (\text{№}113) \begin{cases} 8 \cdot x_1 + 5 \cdot x_2 + 4 \cdot x_3 + 3 \cdot x_4 = 6 \\ 4 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 + 2 \cdot x_4 = 5 \\ 2 \cdot x_1 + 1 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4 = 2 \\ 1 \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + 1 \cdot x_4 = 2 \end{cases}$$

Variant 3.

Solve the systems of simple equations

$$1. (\text{№}116) \begin{cases} 1 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 4 \\ 2 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 = 3 \\ 1 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 = 5 \end{cases}$$

$$2. (\text{№}104) \begin{cases} 2 \cdot x_1 + 3 \cdot x_2 - 1 \cdot x_3 + 4 \cdot x_4 = 0 \\ 1 \cdot x_1 + 5 \cdot x_2 - 4 \cdot x_3 + 9 \cdot x_4 = 0 \\ 3 \cdot x_1 - 1 \cdot x_2 + 4 \cdot x_3 - 5 \cdot x_4 = 0 \\ 1 \cdot x_1 + 2 \cdot x_2 - 1 \cdot x_3 + 3 \cdot x_4 = 0 \end{cases}$$

$$3. (\text{№}90) \begin{cases} 2 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 - 2 \cdot x_4 + 1 \cdot x_5 = 0 \\ 1 \cdot x_1 + 1 \cdot x_2 - 1 \cdot x_3 - 1 \cdot x_4 + 0 \cdot x_5 = 0 \\ 2 \cdot x_1 + 3 \cdot x_2 + 1 \cdot x_3 + 0 \cdot x_4 - 3 \cdot x_5 = 0 \\ 1 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 - 2 \cdot x_4 + 3 \cdot x_5 = 0 \\ 1 \cdot x_1 + 2 \cdot x_2 + 4 \cdot x_3 - 2 \cdot x_4 + 3 \cdot x_5 = 0 \end{cases}$$

$$4. (\text{№}100) \begin{cases} 3 \cdot x_1 - 1 \cdot x_2 - 7 \cdot x_3 - 1 \cdot x_4 + 5 \cdot x_5 = 3 \\ 2 \cdot x_1 + 3 \cdot x_2 - 1 \cdot x_3 - 8 \cdot x_4 - 4 \cdot x_5 = -20 \\ 4 \cdot x_1 - 1 \cdot x_2 - 9 \cdot x_3 - 2 \cdot x_4 + 6 \cdot x_5 = 2 \\ 2 \cdot x_1 - 5 \cdot x_2 - 9 \cdot x_3 + 8 \cdot x_4 + 12 \cdot x_5 = 28 \\ 1 \cdot x_1 + 2 \cdot x_2 + 0 \cdot x_3 - 5 \cdot x_4 - 3 \cdot x_5 = -13 \end{cases}$$

$$5. (\text{№}107) \begin{cases} 2 \cdot x_1 + 5 \cdot x_2 + 7 \cdot x_3 + 1 \cdot x_4 = 7 \\ 3 \cdot x_1 + 2 \cdot x_2 + 5 \cdot x_3 - 4 \cdot x_4 = 16 \\ 2 \cdot x_1 + 3 \cdot x_2 + 5 \cdot x_3 - 1 \cdot x_4 = 9 \\ 1 \cdot x_1 + 5 \cdot x_2 + 6 \cdot x_3 + 3 \cdot x_4 = 1 \end{cases}$$

Variant 4.

Solve the systems of simple equations

$$1. (\text{№}117) \begin{cases} 1 \cdot x_1 - 1 \cdot x_2 + 2 \cdot x_3 = 9 \\ 1 \cdot x_1 + 2 \cdot x_2 + 4 \cdot x_3 = -4 \\ 1 \cdot x_1 + 3 \cdot x_2 + 9 \cdot x_3 = -4 \end{cases}$$

$$\begin{aligned}
2. (\text{№}105) & \begin{cases} 3 \cdot x_1 + 1 \cdot x_2 + 4 \cdot x_3 - 5 \cdot x_4 = 13 \\ 2 \cdot x_1 - 5 \cdot x_2 - 3 \cdot x_3 + 8 \cdot x_4 = 3 \\ 4 \cdot x_1 + 3 \cdot x_2 + 7 \cdot x_3 - 10 \cdot x_4 = 19 \\ 1 \cdot x_1 + 3 \cdot x_2 + 4 \cdot x_3 - 7 \cdot x_4 = 7 \end{cases} \\
3. (\text{№}91) & \begin{cases} 1 \cdot x_1 + 3 \cdot x_2 + 3 \cdot x_3 - 4 \cdot x_4 + 2 \cdot x_5 = 0 \\ 1 \cdot x_1 + 1 \cdot x_2 + 2 \cdot x_3 - 3 \cdot x_4 + 1 \cdot x_5 = 0 \\ 3 \cdot x_1 + 1 \cdot x_2 + 6 \cdot x_3 - 9 \cdot x_4 + 3 \cdot x_5 = 0 \\ 1 \cdot x_1 - 3 \cdot x_2 + 1 \cdot x_3 - 2 \cdot x_4 + 0 \cdot x_5 = 0 \\ 2 \cdot x_1 - 2 \cdot x_2 + 3 \cdot x_3 - 4 \cdot x_4 + 3 \cdot x_5 = 0 \end{cases} \\
4. (\text{№}101) & \begin{cases} 3 \cdot x_1 - 4 \cdot x_2 + 1 \cdot x_3 + 11 \cdot x_4 - 10 \cdot x_5 = 7 \\ 2 \cdot x_1 + 3 \cdot x_2 - 5 \cdot x_3 - 4 \cdot x_4 - 1 \cdot x_5 = -1 \\ 1 \cdot x_1 - 2 \cdot x_2 + 1 \cdot x_3 + 5 \cdot x_4 - 4 \cdot x_5 = 3 \\ 4 \cdot x_1 + 3 \cdot x_2 - 7 \cdot x_3 - 2 \cdot x_4 - 5 \cdot x_5 = 1 \\ 2 \cdot x_1 - 3 \cdot x_2 + 1 \cdot x_3 + 8 \cdot x_4 - 7 \cdot x_5 = 5 \end{cases} \\
5. (\text{№}103) & \begin{cases} 3 \cdot x_1 + 1 \cdot x_2 + 1 \cdot x_3 - 7 \cdot x_4 = 0 \\ 4 \cdot x_1 - 3 \cdot x_2 + 10 \cdot x_3 - 5 \cdot x_4 = 0 \\ 2 \cdot x_1 + 3 \cdot x_2 - 4 \cdot x_3 - 7 \cdot x_4 = 0 \\ 1 \cdot x_1 + 4 \cdot x_2 - 7 \cdot x_3 - 6 \cdot x_4 = 0 \end{cases}
\end{aligned}$$

Variant 5.

Solve the systems of simple equations

$$\begin{aligned}
1. (\text{№}118) & \begin{cases} 1 \cdot x_1 - 2 \cdot x_2 + 1 \cdot x_3 = 0 \\ 2 \cdot x_1 - 1 \cdot x_2 + 1 \cdot x_3 = 0 \\ 1 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 = 0 \\ 2 \cdot x_1 + 3 \cdot x_2 + 1 \cdot x_3 = 0 \end{cases} \\
2. (\text{№}106) & \begin{cases} 5 \cdot x_1 + 2 \cdot x_2 - 1 \cdot x_3 - 8 \cdot x_4 = -22 \\ 3 \cdot x_1 + 1 \cdot x_2 - 1 \cdot x_3 - 5 \cdot x_4 = -13 \\ 1 \cdot x_1 + 4 \cdot x_2 + 7 \cdot x_3 + 2 \cdot x_4 = -8 \\ 2 \cdot x_1 + 9 \cdot x_2 + 16 \cdot x_3 + 5 \cdot x_4 = -17 \end{cases} \\
3. (\text{№}92) & \begin{cases} 2 \cdot x_1 + 1 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 - 2 \cdot x_5 = 0 \\ 2 \cdot x_1 + 6 \cdot x_2 + 3 \cdot x_3 + 5 \cdot x_4 - 4 \cdot x_5 = 0 \\ 1 \cdot x_1 + 1 \cdot x_2 + 3 \cdot x_3 - 2 \cdot x_4 + 1 \cdot x_5 = 0 \\ 1 \cdot x_1 + 1 \cdot x_2 + 2 \cdot x_3 + 1 \cdot x_4 - 1 \cdot x_5 = 0 \\ 1 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 + 0 \cdot x_4 - 2 \cdot x_5 = 0 \end{cases} \\
4. (\text{№}96) & \begin{cases} 2 \cdot x_1 + 1 \cdot x_2 - 2 \cdot x_3 + 3 \cdot x_4 + 6 \cdot x_5 = 7 \\ 1 \cdot x_1 - 1 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 - 4 \cdot x_5 = -7 \\ 1 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 - 1 \cdot x_4 + 3 \cdot x_5 = 9 \\ 3 \cdot x_1 + 2 \cdot x_2 + 4 \cdot x_3 + 1 \cdot x_4 + 3 \cdot x_5 = 9 \\ 2 \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + 5 \cdot x_4 + 4 \cdot x_5 = 3 \\ 2 \cdot x_1 + 3 \cdot x_2 + 4 \cdot x_3 - 3 \cdot x_4 + 4 \cdot x_5 = 15 \end{cases}
\end{aligned}$$

$$5. (\text{№}104) \begin{cases} 2 \cdot x_1 + 3 \cdot x_2 - 1 \cdot x_3 + 4 \cdot x_4 = 0 \\ 1 \cdot x_1 + 5 \cdot x_2 - 4 \cdot x_3 + 9 \cdot x_4 = 0 \\ 3 \cdot x_1 - 1 \cdot x_2 + 4 \cdot x_3 - 5 \cdot x_4 = 0 \\ 1 \cdot x_1 + 2 \cdot x_2 - 1 \cdot x_3 + 3 \cdot x_4 = 0 \end{cases}$$

Variant 6.

Solve the systems of simple equations

$$1. (\text{№}113) \begin{cases} 8 \cdot x_1 + 5 \cdot x_2 + 4 \cdot x_3 + 3 \cdot x_4 = 6 \\ 4 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 + 2 \cdot x_4 = 5 \\ 2 \cdot x_1 + 1 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4 = 2 \\ 1 \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + 1 \cdot x_4 = 2 \end{cases}$$

$$2. (\text{№}107) \begin{cases} 2 \cdot x_1 + 5 \cdot x_2 + 7 \cdot x_3 + 1 \cdot x_4 = 7 \\ 3 \cdot x_1 + 2 \cdot x_2 + 5 \cdot x_3 - 4 \cdot x_4 = 16 \\ 2 \cdot x_1 + 3 \cdot x_2 + 5 \cdot x_3 - 1 \cdot x_4 = 9 \\ 1 \cdot x_1 + 5 \cdot x_2 + 6 \cdot x_3 + 3 \cdot x_4 = 1 \end{cases}$$

$$3. (\text{№}93) \begin{cases} 2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 + 5 \cdot x_4 - 9 \cdot x_5 = 0 \\ 1 \cdot x_1 - 3 \cdot x_2 + 2 \cdot x_3 + 7 \cdot x_4 - 10 \cdot x_5 = 0 \\ 2 \cdot x_1 + 5 \cdot x_2 + 2 \cdot x_3 + 1 \cdot x_4 - 3 \cdot x_5 = 0 \\ 0 \cdot x_1 + 2 \cdot x_2 + 1 \cdot x_3 - 1 \cdot x_4 - 1 \cdot x_5 = 0 \\ 1 \cdot x_1 + 1 \cdot x_2 + 2 \cdot x_3 + 3 \cdot x_4 - 6 \cdot x_5 = 0 \end{cases}$$

$$4. (\text{№}95) \begin{cases} 1 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 + 1 \cdot x_4 - 3 \cdot x_5 = 0 \\ 2 \cdot x_1 + 1 \cdot x_2 - 2 \cdot x_3 - 1 \cdot x_4 - 6 \cdot x_5 = 0 \\ 2 \cdot x_1 + 3 \cdot x_2 + 1 \cdot x_3 + 0 \cdot x_4 - 7 \cdot x_5 = 0 \\ 3 \cdot x_1 - 1 \cdot x_2 - 2 \cdot x_3 + 2 \cdot x_4 - 3 \cdot x_5 = 0 \\ 0 \cdot x_1 + 2 \cdot x_2 - 3 \cdot x_3 - 5 \cdot x_4 - 7 \cdot x_5 = 0 \end{cases}$$

$$5. (\text{№}112) \begin{cases} 2 \cdot x_1 + 1 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 = 1 \\ 3 \cdot x_1 - 2 \cdot x_2 + 1 \cdot x_3 + 4 \cdot x_4 = -4 \\ 1 \cdot x_1 + 3 \cdot x_2 - 2 \cdot x_3 + 3 \cdot x_4 = 5 \\ 2 \cdot x_1 + 3 \cdot x_2 - 2 \cdot x_3 + 1 \cdot x_4 = 12 \end{cases}$$

Variant 7.

Solve the systems of simple equations

$$1. (\text{№}114) \begin{cases} 1 \cdot x_1 + 4 \cdot x_2 + 1 \cdot x_3 = 12 \\ -3 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 = 2 \\ 3 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 = 3 \end{cases}$$

$$2. (\text{№}108) \begin{cases} 3 \cdot x_1 + 4 \cdot x_2 + 2 \cdot x_3 - 1 \cdot x_4 = -3 \\ 5 \cdot x_1 - 2 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 = 1 \\ 2 \cdot x_1 + 7 \cdot x_2 - 4 \cdot x_3 + 3 \cdot x_4 = 5 \\ 1 \cdot x_1 - 3 \cdot x_2 + 6 \cdot x_3 - 4 \cdot x_4 = 2 \end{cases}$$

$$\begin{aligned}
3. (\text{№}94) & \begin{cases} 3 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 + 10 \cdot x_4 - 1 \cdot x_5 = 0 \\ 1 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 + 6 \cdot x_4 + 1 \cdot x_5 = 0 \\ 2 \cdot x_1 + 1 \cdot x_2 + 1 \cdot x_3 + 6 \cdot x_4 - 1 \cdot x_5 = 0 \\ 2 \cdot x_1 + 3 \cdot x_2 - 5 \cdot x_3 + 2 \cdot x_4 + 17 \cdot x_5 = 0 \\ 1 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 + 9 \cdot x_4 + 5 \cdot x_5 = 0 \\ 1 \cdot x_1 - 1 \cdot x_2 + 0 \cdot x_3 + 1 \cdot x_4 - 4 \cdot x_5 = 0 \end{cases} \\
4. (\text{№}98) & \begin{cases} 1 \cdot x_1 + 1 \cdot x_2 + 2 \cdot x_3 + 3 \cdot x_4 + 3 \cdot x_5 = 3 \\ 2 \cdot x_1 + 3 \cdot x_2 - 1 \cdot x_3 - 2 \cdot x_4 - 7 \cdot x_5 = -1 \\ 2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 + 1 \cdot x_5 = 2 \\ 1 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 + 3 \cdot x_5 = 3 \\ 3 \cdot x_1 - 1 \cdot x_2 + 0 \cdot x_3 + 4 \cdot x_4 - 2 \cdot x_5 = 0 \end{cases} \\
5. (\text{№}111) & \begin{cases} 4 \cdot x_1 + 5 \cdot x_2 + 4 \cdot x_3 - 2 \cdot x_4 = 4 \\ 2 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 - 1 \cdot x_4 = 2 \\ 3 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 - 2 \cdot x_4 = -6 \\ -2 \cdot x_1 - 1 \cdot x_2 - 1 \cdot x_3 + 1 \cdot x_4 = 1 \end{cases}
\end{aligned}$$

Variant 8.

Solve the systems of simple equations

$$\begin{aligned}
1. (\text{№}116) & \begin{cases} 1 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 4 \\ 2 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 = 3 \\ 1 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 = 5 \end{cases} \\
2. (\text{№}109) & \begin{cases} 1 \cdot x_1 + 2 \cdot x_2 + 1 \cdot x_3 - 3 \cdot x_4 = 4 \\ 2 \cdot x_1 - 5 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 = -6 \\ 4 \cdot x_1 + 1 \cdot x_2 + 2 \cdot x_3 - 1 \cdot x_4 = 3 \\ 2 \cdot x_1 + 6 \cdot x_2 - 1 \cdot x_3 - 3 \cdot x_4 = 5 \end{cases} \\
3. (\text{№}95) & \begin{cases} 1 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 + 1 \cdot x_4 - 3 \cdot x_5 = 0 \\ 2 \cdot x_1 + 1 \cdot x_2 - 2 \cdot x_3 - 1 \cdot x_4 - 6 \cdot x_5 = 0 \\ 2 \cdot x_1 + 3 \cdot x_2 + 1 \cdot x_3 + 0 \cdot x_4 - 7 \cdot x_5 = 0 \\ 3 \cdot x_1 - 1 \cdot x_2 - 2 \cdot x_3 + 2 \cdot x_4 - 3 \cdot x_5 = 0 \\ 0 \cdot x_1 + 2 \cdot x_2 - 3 \cdot x_3 - 5 \cdot x_4 - 7 \cdot x_5 = 0 \end{cases} \\
4. (\text{№}97) & \begin{cases} 2 \cdot x_1 - 2 \cdot x_2 + 3 \cdot x_3 + 1 \cdot x_4 + 6 \cdot x_5 = 5 \\ 1 \cdot x_1 + 2 \cdot x_2 + 1 \cdot x_3 + 2 \cdot x_4 - 5 \cdot x_5 = -2 \\ 3 \cdot x_1 + 1 \cdot x_2 + 4 \cdot x_3 + 2 \cdot x_4 + 0 \cdot x_5 = 3 \\ 2 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 - 3 \cdot x_4 - 1 \cdot x_5 = 4 \\ 1 \cdot x_1 - 3 \cdot x_2 - 1 \cdot x_3 + 2 \cdot x_4 + 3 \cdot x_5 = 3 \end{cases} \\
5. (\text{№}106) & \begin{cases} 5 \cdot x_1 + 2 \cdot x_2 - 1 \cdot x_3 - 8 \cdot x_4 = -22 \\ 3 \cdot x_1 + 1 \cdot x_2 - 1 \cdot x_3 - 5 \cdot x_4 = -13 \\ 1 \cdot x_1 + 4 \cdot x_2 + 7 \cdot x_3 + 2 \cdot x_4 = -8 \\ 2 \cdot x_1 + 9 \cdot x_2 + 16 \cdot x_3 + 5 \cdot x_4 = -17 \end{cases}
\end{aligned}$$

Variant 9.

Solve the systems of simple equations

$$\begin{aligned}
1. (\text{№117}) & \begin{cases} 1 \cdot x_1 - 1 \cdot x_2 + 2 \cdot x_3 = 9 \\ 1 \cdot x_1 + 2 \cdot x_2 + 4 \cdot x_3 = -4 \\ 1 \cdot x_1 + 3 \cdot x_2 + 9 \cdot x_3 = -4 \end{cases} \\
2. (\text{№110}) & \begin{cases} 3 \cdot x_1 - 2 \cdot x_2 + 2 \cdot x_3 + 5 \cdot x_4 = 4 \\ 2 \cdot x_1 + 4 \cdot x_2 - 1 \cdot x_3 + 3 \cdot x_4 = 0 \\ 2 \cdot x_1 - 3 \cdot x_2 + 3 \cdot x_3 + 4 \cdot x_4 = 3 \\ 5 \cdot x_1 + 2 \cdot x_2 + 1 \cdot x_3 + 8 \cdot x_4 = -1 \end{cases} \\
3. (\text{№96}) & \begin{cases} 2 \cdot x_1 + 1 \cdot x_2 - 2 \cdot x_3 + 3 \cdot x_4 + 6 \cdot x_5 = 7 \\ 1 \cdot x_1 - 1 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 - 4 \cdot x_5 = -7 \\ 1 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 - 1 \cdot x_4 + 3 \cdot x_5 = 9 \\ 3 \cdot x_1 + 2 \cdot x_2 + 4 \cdot x_3 + 1 \cdot x_4 + 3 \cdot x_5 = 9 \\ 2 \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + 5 \cdot x_4 + 4 \cdot x_5 = 3 \\ 2 \cdot x_1 + 3 \cdot x_2 + 4 \cdot x_3 - 3 \cdot x_4 + 4 \cdot x_5 = 15 \end{cases} \\
4. (\text{№100}) & \begin{cases} 3 \cdot x_1 - 1 \cdot x_2 - 7 \cdot x_3 - 1 \cdot x_4 + 5 \cdot x_5 = 3 \\ 2 \cdot x_1 + 3 \cdot x_2 - 1 \cdot x_3 - 8 \cdot x_4 - 4 \cdot x_5 = -20 \\ 4 \cdot x_1 - 1 \cdot x_2 - 9 \cdot x_3 - 2 \cdot x_4 + 6 \cdot x_5 = 2 \\ 2 \cdot x_1 - 5 \cdot x_2 - 9 \cdot x_3 + 8 \cdot x_4 + 12 \cdot x_5 = 28 \\ 1 \cdot x_1 + 2 \cdot x_2 + 0 \cdot x_3 - 5 \cdot x_4 - 3 \cdot x_5 = -13 \end{cases} \\
5. (\text{№102}) & \begin{cases} 1 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 - 3 \cdot x_4 = 0 \\ 4 \cdot x_1 - 1 \cdot x_2 - 5 \cdot x_3 + 14 \cdot x_4 = 0 \\ 2 \cdot x_1 + 5 \cdot x_2 + 3 \cdot x_3 - 4 \cdot x_4 = 0 \\ 3 \cdot x_1 + 2 \cdot x_2 - 1 \cdot x_3 + 5 \cdot x_4 = 0 \end{cases}
\end{aligned}$$

Variant 4.

Solve the systems of simple equations

$$\begin{aligned}
1. (\text{№118}) & \begin{cases} 1 \cdot x_1 - 2 \cdot x_2 + 1 \cdot x_3 = 0 \\ 2 \cdot x_1 - 1 \cdot x_2 + 1 \cdot x_3 = 0 \\ 1 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 = 0 \\ 2 \cdot x_1 + 3 \cdot x_2 + 1 \cdot x_3 = 0 \end{cases} \\
2. (\text{№111}) & \begin{cases} 4 \cdot x_1 + 5 \cdot x_2 + 4 \cdot x_3 - 2 \cdot x_4 = 4 \\ 2 \cdot x_1 + 2 \cdot x_2 + 2 \cdot x_3 - 1 \cdot x_4 = 2 \\ 3 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 - 2 \cdot x_4 = -6 \\ -2 \cdot x_1 - 1 \cdot x_2 - 1 \cdot x_3 + 1 \cdot x_4 = 1 \end{cases} \\
3. (\text{№97}) & \begin{cases} 2 \cdot x_1 - 2 \cdot x_2 + 3 \cdot x_3 + 1 \cdot x_4 + 6 \cdot x_5 = 5 \\ 1 \cdot x_1 + 2 \cdot x_2 + 1 \cdot x_3 + 2 \cdot x_4 - 5 \cdot x_5 = -2 \\ 3 \cdot x_1 + 1 \cdot x_2 + 4 \cdot x_3 + 2 \cdot x_4 + 0 \cdot x_5 = 3 \\ 2 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 - 3 \cdot x_4 - 1 \cdot x_5 = 4 \\ 1 \cdot x_1 - 3 \cdot x_2 - 1 \cdot x_3 + 2 \cdot x_4 + 3 \cdot x_5 = 3 \end{cases}
\end{aligned}$$

$$4. (\text{№}99) \begin{cases} 1 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 - 4 \cdot x_4 - 1 \cdot x_5 = -2 \\ 2 \cdot x_1 - 1 \cdot x_2 + 3 \cdot x_3 + 0 \cdot x_4 - 1 \cdot x_5 = -1 \\ 4 \cdot x_1 + 2 \cdot x_2 - 1 \cdot x_3 + 3 \cdot x_4 + 5 \cdot x_5 = 19 \\ 3 \cdot x_1 - 2 \cdot x_2 + 3 \cdot x_3 + 2 \cdot x_4 + 0 \cdot x_5 = 3 \\ 2 \cdot x_1 + 1 \cdot x_2 + 4 \cdot x_3 - 3 \cdot x_4 - 2 \cdot x_5 = -4 \end{cases}$$

$$5. (\text{№}105) \begin{cases} 3 \cdot x_1 + 1 \cdot x_2 + 4 \cdot x_3 - 5 \cdot x_4 = 13 \\ 2 \cdot x_1 - 5 \cdot x_2 - 3 \cdot x_3 + 8 \cdot x_4 = 3 \\ 4 \cdot x_1 + 3 \cdot x_2 + 7 \cdot x_3 - 10 \cdot x_4 = 19 \\ 1 \cdot x_1 + 3 \cdot x_2 + 4 \cdot x_3 - 7 \cdot x_4 = 7 \end{cases}$$